Ministry of Finance of the Czech Republic

RESEARCH STUDY

The estimation of the NAIRU for the Czech economy

Vít Pošta

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// Abstrakt

Příspěvek se zabývá modelováním NAIRU pro českou ekonomiku. V příspěvku jsou zvoleny dvě metody odhadu NAIRU. První metoda, jež byla převzata od OECD, a která je Evropskou komisí doporučována pro modelování NAIRU v nově přistupivších zemích, představuje základ strukturálních modelů NAIRU. Druhá metoda, zcela jasně převažující pro modelování NAIRU, vychází z Gordonova modelu trojúhelníku a samotné NAIRU modeluje jako autoregresní proces. Pro konečné řešení modelu je využit Kalmanův filtr.

Klíčová slova: Gordonův model trojúhelníku, inflace, Kalmanův filtr, NAIRU, strukturální model.

Abstract

The paper deals with NAIRU modelling in the context of the Czech economy. There are two methods used in the paper. The first one, developed by OECD, and recommended by the European Commission for NAIRU modelling in the new Member States, represents the basis of the structural models approach. The second one, a dominating method, builds on the Gordon triangle model and the NAIRU itself is modelled as an autoregressive process. Kalman filtering procedure is used to solve the model.

Keywords: Gordon triangle model, inflation, Kalman filter, the NAIRU, structural model.

JEL Classification: C13, C22, E24.

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The purpose of this paper is the analysis of the empirical estimation of the NAIRU for the Czech economy. The paper does not deal with theoretical definition of the term itself and its knowledge is taken for granted.

The whole paper is divided into three sections. The first section briefly presents the approaches used for the estimation of the NAIRU. The second section discusses the OECD model that can be considered to be the basic structural model of the NAIRU. This model is first derived and then used for the estimation of the NAIRU in the environment of the Czech economy. The third section, on the other hand, is based on statistical approach towards the estimation of the NAIRU in the environment of the Czech economy. The Gordon triangle model which reflects inflation inertia, the output gap and supply shocks is taken as its basis. The formulated model is resolved by means of the Kalman filter. The Kalman filter itself is briefly presented in the Appendix.

The text presents the results of estimates in graphs, specific output figures are given in Annex 1.

1 NAIRU estimation methods

The first important question associated with the estimation of the NAIRU, is the awareness of what particular NAIRU we intend to estimate. At the empirical level, three possible estimates of the NAIRU are distinguished, e.g. Richardson (2000):

a) short-term,
b) medium-term, „core“ NAIRU and
c) long-term

The short-term NAIRU is such rate of unemployment, for which the current rate of inflation can be fixed also for the subsequent period. However, this means that such rate of unemployment, and hence the NAIRU, is relatively quite variable, since it is affected by all supply shocks and inflation inertia. Hence, this estimation takes no account whatsoever of the system adaptation to at least short-term supply shocks.

The phenomenon, that is usually taken to be the NAIRU, is the so-called medium-term NAIRU. This is an equilibrium parameter to which the actual rate of unemployment tends to develop after temporary supply shocks subside and the actual rate of inflation adapts to its expected value. The approach towards the NAIRU presented in this way makes it quite obviously subject to the augmented Phillips curve.

The long-term NAIRU is the equilibrium rate of unemployment to which the system tends to develop after all supply (and other) shocks subside, including those that have long-term effect on the economy. Richardson (2000, p.8) states literally „similarly as the natural rate (*of unemployment*, *the author's note*)“. However, this proposition can be questioned. The difference between the NAIRU and the natural rate of unemployment consists in particular in their macroeconomic or microeconomic basis and although under certain circumstances it might be considered to be identical, in empirical, theoretical (and interpretational) terms, these are always different parameters.

The medium-term NAIRU can fully rely on the initial Gordon triangle model, Gordon (1997), which has been taken as the basis for the model that will be presented in section 3. Now let me comment on the NAIRU estimation methods.

In professional literature, usually three groups of NAIRU estimation models are defined, e.g. Szeto and Guy (2004):

a) structural models,
b) purely statistical models and
c) reduced form models.

The core of the structural models fully builds on the Phillips curve model. To put it more precisely, it builds on its background which consists in the structure of the functions of the aggregate wage rate and the aggregate price level. These functions establish the relationship between the wage rate and factors that affect it (apart from the expected price level) from the viewpoint of employees and the relationship between the price level and factors that affect it (apart from the aggregate wage rate) from the viewpoint of companies. The rate of unemployment that is consistent with those two functions and for which the price level is stable is the NAIRU we are looking for.

The advantages of this approach consist quite obviously in the fact that it is possible to incorporate into the model a large number of purely economic factors and circumstances, including a number of specific features of a particular economy. On the other hand, more analytical approach requires that the functions of the aggregate wage rate and the aggregate price level reflect the factors that are very hard to be measured and identified. The second problematic aspect stems from the fact that due to heavy reliance of these parameters on the economic theory, they require the acceptance of a number of assumptions pertaining to the behaviour of economic agents on which the views differ significantly. These models and their results are then necessarily affected by subjectivity stemming from this situation.

The purely statistical models are usually based only on the actual rate of unemployment and by means of the chosen filter (e.g. the often used Hodrick-Prescott filter) they estimate the trend that is considered to be the NAIRU being sought. Obviously, the ease and speed of calculation are their advantage. Their two key weaknesses include subjectivity concerning the parameters of a given filter that need to be chosen, and in particular the fact that filters, which are based on the moving average principle, absorb slowly significant changes in the rate of unemployment that can be structural in nature. Generally, it holds that this approach makes impossible the use of any economic considerations.

The last group of the reduced form models combines the previous two groups of models. Usually, it is based on the Phillips curve model, specifically the Gordon triangle model and by means of selected statistical methods estimates the unobservable NAIRU. The most frequently used statistical methods include the models making use of the Kalman filter, the structural VAR models (SVAR) and VEC models for their solutions. It needs to be noted that these methods alone do not give any economic insight into the problem. However, the structure of the model, for whose solution these methods are used, allows to incorporate into it the relevant economic circumstances.

Initially, a simple structural model is described and applied. This is virtually only a preliminary model and it definitely does not require any searching for or estimation of qualitative characteristics of the labour market, etc. This is only a trivial labour market model. Moreover, at the last stage of the calculation, the Hodrick-Prescott filter is used which somewhat complicates its inclusion among the first group of models.

Reduced form models play obviously a dominant role in the estimation of the NAIRU. The Kalman filter is by far the most widely used tool for the estimation of the NAIRU. This model is discussed subsequently in the third section of this paper.

2 Initial OECD structural model

In the first part of this chapter, the principle of the model and its derivation are generally defined fully in compliance with ECFIN/475/03-EN. This model was at the time of its formulation applied to the calculation of the NAIRU for the purposes of calculation of the output gap for the newly acceding countries by DG ECFIN. At the same time, also the manner of its practical calculation is outlined. The second part presents the estimates of the NAIRU that have been calculated on the basis of this procedure.

2.1 Structure of the model

As has been noted above, this is the trivial labour market model. Consequently, let me define two functions: the so-called wage rule function and the demand for labour function. The wage rule function is based on the supply of labour and establishes the relationship between the development of the expected real wage rate on one hand and the labour productivity and the unemployment gap on the other:

$$w_t - p_t^e = (y_t - l_t) - \beta(u_t - nairu_t) + u_t^w, \tag{2.1}$$

where w is the year-on-year average nominal wage index[1], p^e is the expected price level index, y is the year-on-year real gross value added index, l is the year-on-year employment index, u is the rate of unemployment[2], β stands for sensitivity of the expected real wage to the unemployment gap and u^w is the stochastic element relating to other possible shocks. The subscript t designates the period. We will subtract from each side of the equation (2.1) the remainder between the year-on-year nominal wage index and the price level index for the previous period:

$$\Delta w_t - \pi_t^e = (y_t - l_t) - (w_{t-1} - p_{t-1}) - \beta(u_t - nairu_t) + u_t^w, \tag{2.2}$$

where Δ is the first difference of the relevant variable and π is the rate of inflation[3].

The second function is the demand for labour which will be expressed as follows:

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[1] This is a figure from national accounts showing the quotient of the aggregate volume of wages and salaries and the total number of employees.

[2] According to the Labour Force Sample Survey (LFSS) methodology.

[3] In factual terms, this concerns the annual rate of inflation calculated on the basis of the consumer price index. However, the data on the rate of inflation will not be required for calculation purposes in this model.

$$w_t - p_t = (y_t - l_t) + u_t^l, \tag{2.3}$$

where u^l stands for the stochastic element. Hence, the development of the real macroeconomic wage is based on the development of the macroeconomic productivity of labour. Let us suppose the adaptive formation of the expected rate of inflation based on the following relationship:

$$\pi_t^e = \pi_{t-1}^e + j(\pi_{t-1} - \pi_{t-1}^e). \tag{2.4}$$

The equation (2.4) says that the current expected rate of inflation is based on the expected rate of inflation for the previous period and the level of reflection of the previous error (remainder between the actual and expected rate of inflation in the previous period) where j is the coefficient which designates the speed of adaptation to the previous error. For the sake of simplicity, we will assume static inflationary expectations which means that we will expect immediate adaptation to the previous error – the coefficient j is equal to one. If at the same time the remainder between the rise in nominal wage in the previous period and the increase in the labour productivity in the previous period is equal to the rate of inflation in the previous period, for the expected rate of inflation in the current period we can write:

$$\pi_t^e = \pi_{t-1} = \Delta w_{t-1} - (\Delta y_{t-1} - \Delta l_{t-1}). \tag{2.5}$$

If we use the relationship (2.5) and equations (2.2) and (2.3), we can express the second difference of the year-on-year nominal wage index:

$$\Delta w_t - \Delta w_{t-1} = (y_t - l_t) - (y_{t-1} - l_{t-1}) - (\Delta y_{t-1} - \Delta l_{t-1}) - \beta(u_t - nairu_t) + \varepsilon_t, \tag{2.6}$$

where ε_t is the remainder between u^w and u^l.

The equation (2.6) can be rewritten as follows:

$$\Delta^2 w_t = (\Delta^2 y_t - \Delta^2 l_t) - \beta(u_t - nairu_t) + \varepsilon_t, \tag{2.7}$$

While apparently it holds that the expression:

$$\Delta^2 w_t - (\Delta^2 y_t - \Delta^2 l_t)$$

represents the second difference of unit labour costs in time t. Hence, finally, it holds that:

$$\Delta^2 ulc_t = -\beta(u_t - nairu_t) + \varepsilon_t, \tag{2.8}$$

where ulc are unit labour costs.

The NAIRU estimation method is very simple in this case. If we assume the constant NAIRU and zero mean value of the stochastic element, we can calculate the beta coefficient from the equation (2.8) by means of the first differentiation with respect to the rate of unemployment:

$$\beta = -\frac{\Delta^3 ulc_t}{\Delta u_t}. \tag{2.9}$$

By means of the calculated beta coefficient we can then calculate from the equation (2.8) the value of the NAIRU, including the stochastic element:

$$nairu_t = u_t + \frac{\Delta^2 ulc}{\beta} + \varepsilon_t. \tag{2.10}$$

The value on the right side of the expression (2.10) is subsequently adjusted for on the basis of the Hodrick-Prescott filter. Thereby the estimate of the NAIRU for the period under review is obtained. It is worthy of mention that with the high level of the beta coefficient, this trivial structural model can be reduced to a purely statistical model, to put in general terms:

$$\lim_{\beta \to \infty} \frac{\Delta^2 ulc}{\beta} = 0 \Rightarrow nairu_t = u_t + \varepsilon_t . \qquad (2.11)$$

From the implication (2.11) is clear that in the case of the high beta coefficient, the NAIRU can be estimated by means of the Hedrick-Prescott filter for the unemployment series which is nothing else but purely statistical approach towards estimation of the NAIRU.

2.2 Estimation of the NAIRU

This section presents specific estimates of the NAIRU for the Czech economy on the basis of the above model. The model derived above will be applied to the quarterly data. In the course of explanation of the model itself it has been specified what data are used for the calculation. Except for the rate of unemployment, which is being examined in accordance with the Labour Force Sample Survey (LFSS) methodology, the data from national accounts are used which are, furthermore, taken as the basis for the routine calculation of the labour productivity, the aggregate volume of wages and salaries for employees and unit labour costs. However, these particular outputs are not used. The source data are taken as the basis for the calculation of the required parameters as outlined above. This means that e.g. labour productivity is not calculated by means of the ratio between the gross value added index and the employment index, but by means of their reminder. At the last stage of the calculation, the Hodrick-Prescott filter was used with the lambda 1600 coefficient.

Figure 1 shows the relationship between the NAIRU and the seasonally adjusted rate of unemployment. The first estimated value of the NAIRU is for the 3rd quarter of 1997, which is caused by the differentiation performed in the course of the calculation in accordance with the above model. The initial value of the NAIRU is 6.3 %. Since the 3rd quarter of 1997 then the value of the NAIRU has been gradually rising. It reached its peak in the 4th quarter of 2001 when the NAIRU was 8.2 %. Subsequently, it has been gradually decreasing to 6.2 % in the 3rd quarter of 2007.

The relationship between the development of the NAIRU and the rate of unemployment since 2006 implies that a sharp fall in the rate of unemployment has been partly caused by a fall in the structural unemployment. Nevertheless, since the 3rd quarter already, the actual rate of unemployment is lower than the estimated NAIRU and this difference is further increased up to 1.0 percentage points in the third quarter of 2007. This fact should be reflected in a fairly significant and positive inflationary output gap. Although moderate inflationary pressures can be observed in the economy, the economy's development over the period under review does not prove much convincingly this development of the NAIRU. The second and much more important question is whether the high estimated rate of growth of the potential output (since the 2nd quarter 2005 over 5 %[4]) should not be reflected more in the fall of the structural unemployment and therefore more rapid decrease of the NAIRU? According to

[4] According to the calculation of the Ministry of Finance of the Czech Republic.

this estimate of the NAIRU, the high rate of growth of the potential output rather seems not to be reflected in its development.

Figure 2 shows the comparison between estimates under this method and purely statistical calculation by means of the H-P filter (1600).

Figure 1: Quarterly NAIRU, OECD method

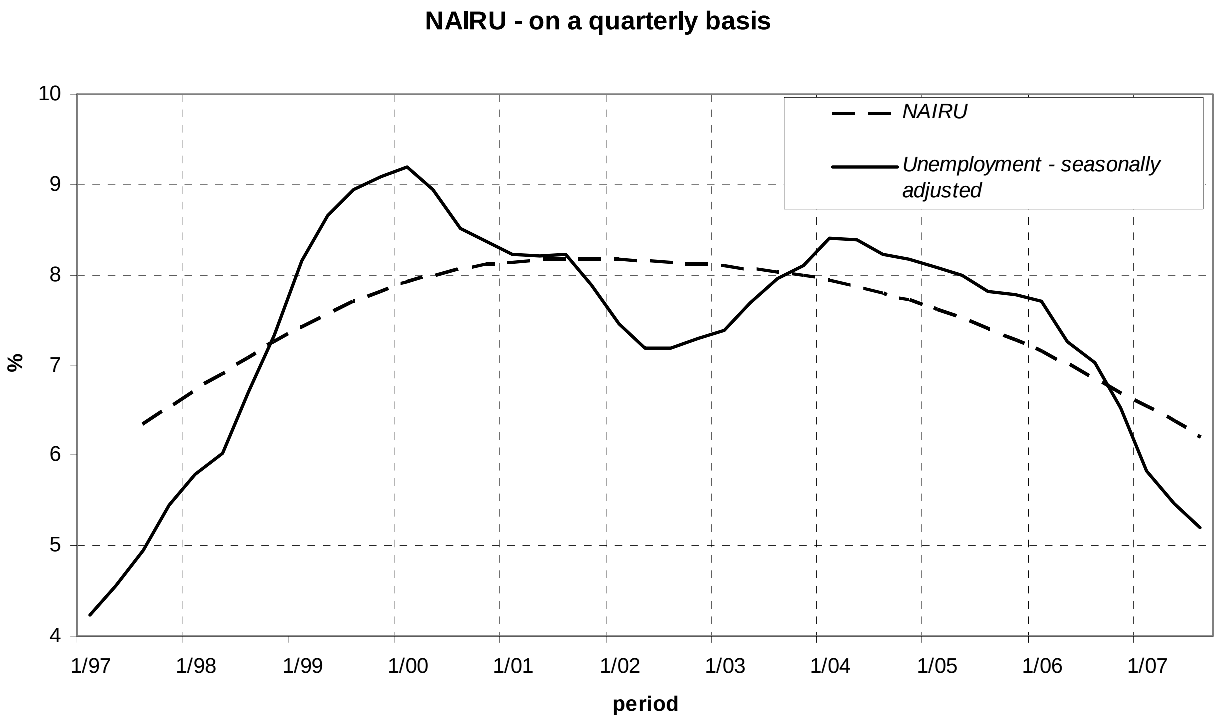
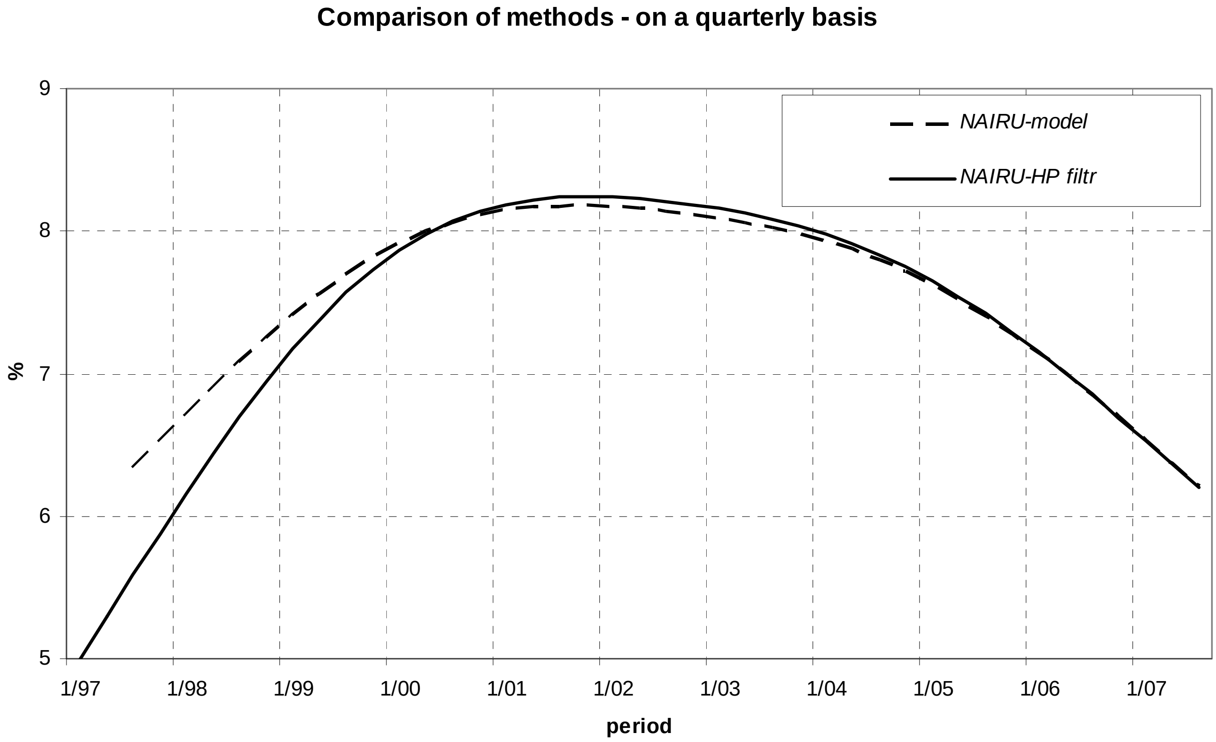


Figure 2 clearly shows that since the 1st quarter of 2005 the results have been almost the same. It stems from the relatively high average beta coefficient for this period: -25.2.

Figure 2: Comparison of the OECD method and H-P filter, quarterly data



3 Model with stochastic differential equations

The purpose of section 3 of this paper is the description and presentation of the key model for the estimation of the NAIRU which uses for its solution the Kalman filter. This model is based on the Gordon triangle model which means that the rate of inflation is modelled with respect to the previous rate of inflation (to put it in more general terms, with respect to the inflationary expectations), the unemployment gap and supply shocks. The NAIRU itself is modelled as an autoregressive process. In the first part, the model will be presented in the general form. Attention will be paid to individual variables and technical aspects of the model. The second part presents the specific form of the model and estimates of the NAIRU for the quarterly data.

3.1 Model

The general form of the model consists of two equations: the first equation describes a variable that is observable (in this context, the rate of inflation) – normally, it is labelled as the measurement equation and the second variable that is not observable (the NAIRU) – which is designated as the transition equation.

The first equation is based on the Gordon triangle model and can be expressed as follows:

$$\begin{aligned}&\pi_t = \alpha(L)\pi_{t-1} - \beta(L)(u_t - nairu_t) + \gamma(L)z_t + \nu_t, \\ &where\ \nu \sim N(0, \sigma_\nu^2)\end{aligned} \qquad (3.1)$$

Hence, the rate of inflation is modelled with respect to the previous rate of inflation, while α(L) stands for polynomial in the delay operator. Consequently, the equation (3.1) shows that the rate of inflation is generally modelled on a random number of its delayed values. This factor expresses the effect of expectations or inflation inertia on the actual rate of inflation. The second factor – the unemployment gap – expresses the effect of the aggregate demand on the actual rate of inflation. Negative value of the unemployment gap indicates excess aggregate demand and hence the inflationary pressures. Similarly, as in previous case β(L) stands for polynomial in the delay operator. The third factor are supply shocks, generally designated as z_t. Account is taken of at least oil prices and the development of the exchange rates as standard factors. The expression γ(L) stands for polynomial in the delay operator. The last term – ν_t – is a stochastic element which has by definition normal distribution with zero mean value and variance σ^2.

The second equation models the estimated NAIRU. In the model presented in this paper, the NAIRU is modelled as the autoregressive process:

$$\begin{aligned}&nairu_t = \delta nairu_{t-1} + \eta_t, \\ &where\ \eta \sim N(0, \sigma_\eta^2)\ a\ \mathrm{cov}(\nu, \eta) = 0\end{aligned} \qquad (3.2)$$

The ratio of variances of stochastic elements:

$$r = \frac{\sigma_\eta^2}{\sigma_\nu^2},$$

plays an important role in the management of the variability of estimates of the NAIRU. The ratio is normally designated as „the signal-to-noise ratio“.

The specific expression of the model will be presented in the following part of this chapter. However, prior to this explanation, I will focus on the general, initial form of the tested model. The following formulation has been used for the initial model:

$$\begin{aligned}&\pi_t = \alpha_1\pi_{t-1} + (1-\alpha_1)\pi_{t-2} + \beta_1(u_t - nairu_t) + \beta_2(u_{t-1} - nairu_{t-1}) + \gamma_1 neer_t + \\ &+ \gamma_2 neer_{t-1} + \gamma_3 neer_{t-2} + \gamma_4 brent_t + \gamma_5 brent_{t-1} + \gamma_6 brent_{t-2} + \gamma_7 pribor_t + \\ &+ \gamma_8 pribor_{t-1} + \gamma_9 pribor_{t-2} + \nu_t, \\ &where\ \nu_t \sim N(0, 0.0625)\end{aligned}, \qquad (3.3)$$

$$\begin{aligned}&nairu_t = \delta_1 nairu_{t-1} + \eta_t, \\ &where\ \eta \sim N(0, 0.05)\end{aligned}. \qquad (3.4)$$

Inflation inertia is reflected in the model as a dependence of the current rate of inflation on the delayed values by one and two periods. In accordance with Gordon (1997), I assume the sum of the relevant coefficients equal to one which guarantees the existence of the long-run vertical Phillips curve. Obviously, it is possible to assume also other formulation of the inflationary expectations than their adaptive formation which is precisely represented by the relationship between the current rate of inflation and the delayed rate of inflation (rate of inflations). However, in an overwhelming majority of cases it is based precisely on the adaptive formation of the inflationary expectations. An exception to this rule is e.g. a study by Hurník and Navrátil (2004) who, apart from the delayed rate of inflation, incorporate into the model the inflationary expectations. These expectations are modelled as a perfect forecast, i.e. the current expectation of the future rate of inflation equals to the actual future rate of inflation. The effect of the demand side is reflected in the model as a dependence of the rate of inflation on the current and by one period delayed remainder between the rate of unemployment and the NAIRU. Supply shocks incorporated into the model include the nominal effective exchange rate which represents the effect of imported goods and services and the Brent oil price. A similar concept of the effect of import prices is used e.g. by Fukač (2003). I also add to the equation for inflation the effect of Pribor (3M Pribor), as an approximation of the effect of the monetary policy on the rate of inflation. Fukač (2003), reflects this effect too, but as part of the equation for the NAIRU. As has been already noted above, the NAIRU is modelled as an autoregressive process. The calibration of variances of random elements is primarily based on Boone (2000) and examines the behaviour of this particular model.

The rate of inflation used in the model is expressed as the annual rate of inflation for the relevant period on the basis of the Consumer Price Index (CPI). The rate of unemployment is taken from the LFSS (Labour Force Sample Survey) statistics. For the nominal exchange rate (NEER) index and the Brent oil price, their natural logarithm is considered. For all indicators, the first differences are computed which in this case result in stationarity of the considered series.

Apart from setting the variations of random elements, also the initial values of the NAIRU and values of the co-variation matrix among unobserved variables – the NAIRU and additional variable are being set. The initial value of the NAIRU is set at 7 %. This is basically an arbitrary choice which, however, need not be far from reality and above all, any acceptable deviation from it, basically, does not affect future estimates. The sensitivity analysis related to the choice of this parameter is set out in Annex 2. The main purpose of the set initial value is to prevent the model from being completely „distant“ from reality. The co-variation matrix is set at high initial values (1,000), which, due to the fact, that the system rapidly departs from any potential inappropriate initial value, accelerates its convergence to an acceptable solution.

The last factor which is being set are initial values of all coefficients which are set at zero.

3.2 Estimated model

The model is assessed in particular by means of statistical significance of the appropriate input parameters that refer to their relevance within the model. The value of the likelihood function and the Akaike's information criterion were used as comparative criteria for choosing between various versions of the model described by equations (3.3) and (3.4).

The output presented in this paper reaches the lowest values of this criterion compared to the options examined. Additional similar criteria having almost the same informative value are as follows: Schwarz criterion and the Hannah-Quinn criterion whose values I do not give in this study. However, even in their case it holds that the models presented here were assessed favourably compared to other tested models.

Now, it is possible to discuss the model for the quarterly NAIRU. Table 1 presents this model. The estimate of the NAIRU is shown in Figure 3 (The estimate has been smoothed by the Hodrick-Prescott (HP) filter with the coefficient 1). The table showing the data is set out in Annex 1.

Table 1: Model of the quarterly NAIRU (* means that the zero hypothesis about insignificance of the given parameter was rejected at the level of significance of 1 %)**

Parameter	Estimate
α_1	0.6040***
β_1	-3.4388***
β_2	3.1320***
γ_1	-29.819***
γ_4	2.9535***
γ_9	-0.4970***
δ_1	0.9978***
Likelihood function	
-105,7995	
AIC	
5,2465	

As far as the estimated values from Table 1 are concerned, parameter α_1 indicates to what extent the current rate of inflation is affected by the rate of inflation delayed by one period. The remainder of this value up to one appertains to the effect of the rate of inflation delayed by two periods. Parameters $\square_1$ and $\square_2$ reflect the effect of the remainder between the rate of unemployment and the NAIRU. It is obvious, that the total effect is negative, i.e. that a fall in the rate of unemployment below the NAIRU level exerts pressure on an increase in the rate of inflation. Parameters $\square_1$, $\square_2$, $\square_3$ reflect the effect of nominal effective exchange rate, Brent oil prices and 3M Pribor. And finally, the estimate of parameter $\square_1$ points to the level of hysteresis in the labour market which is relatively low, according to this estimate.

Estimation of the NAIRU by means of the Kalman filter in comparison with the development of the rate of unemployment indicates that a significant fall in the rate of unemployment since 2006 stems rather from a change in the structural rate of unemployment and only in the recent period (2nd and 3rd quarter of 2007), it is possible to observe clear tendencies towards a fall in the rate of unemployment below the NAIRU level and hence a signal of possible inflationary pressures.

In Figure 4, it is possible to compare estimation methods used here. It is obvious that the estimate of the NAIRU by means of the Kalman filter is more variable than the estimate of the initial structural model. In this context, account needs to be taken of a very strong correlation between the estimate using the initial structural model and the estimate using the HP filter. The higher variability of the NAIRU definitely does not reduce the weight of its estimate and is not inconsistent with the theory. According to Gordon (1997), it is quite possible that the NAIRU shows negligible variability. However, sharp changes between individual quarters should be avoided when producing the estimate which is not the case with the estimate presented in this study.

Figure 3: NAIRU – on a quarterly basis

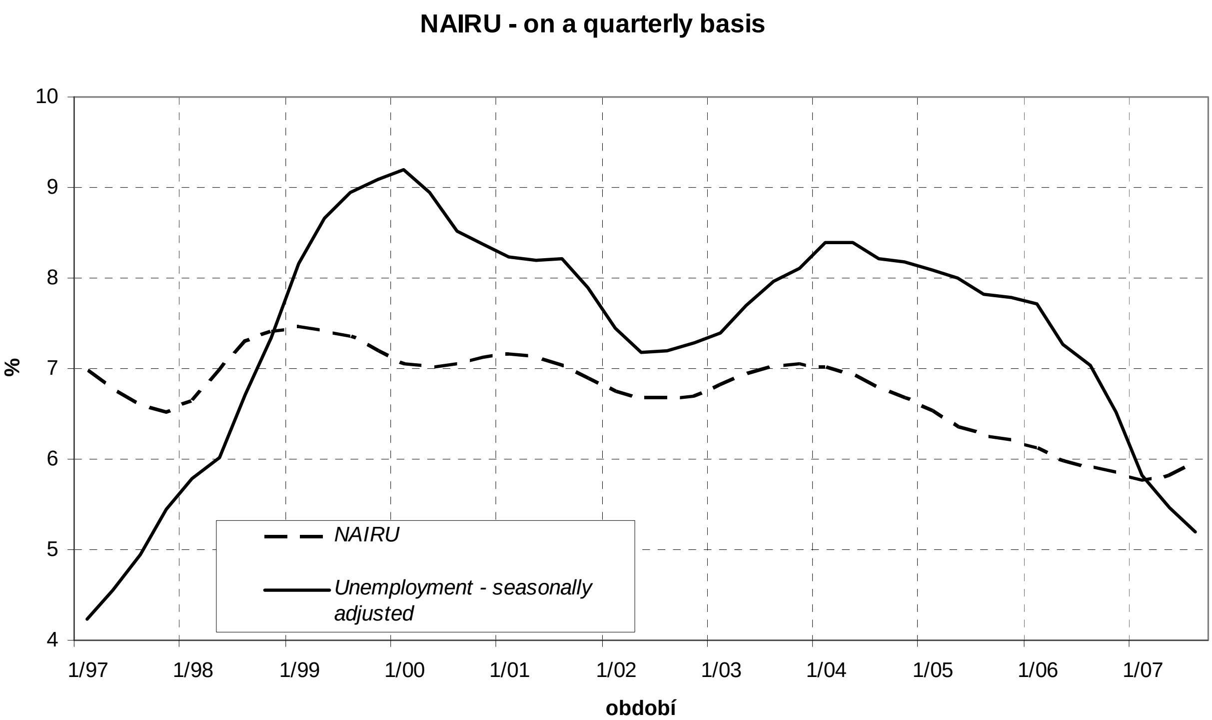
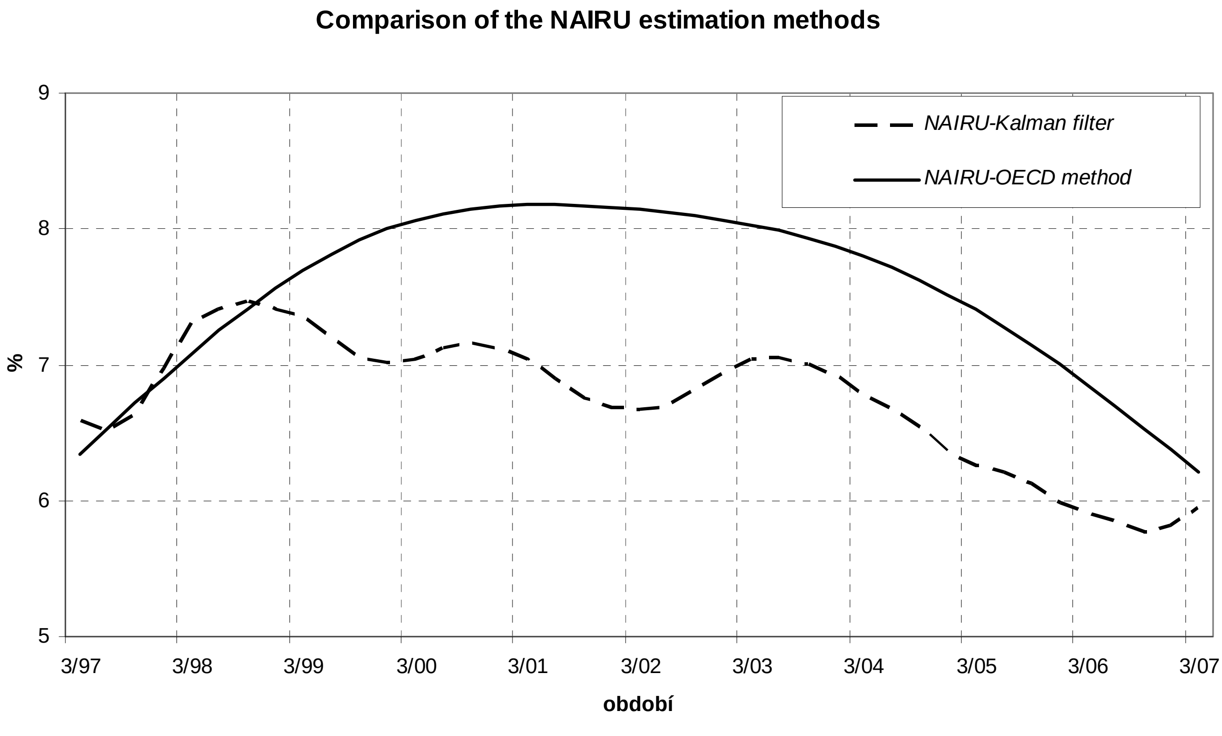


Figure 4: Comparison of methods



Both estimates of the NAIRU presented in this paper clearly show that their amount, except for the economy's most recent development, is relatively high. This is caused in particular by the institutional background of the labour market. The current setting of the social system does not unambiguously promote the preference of work to leisure time. The second important factor were structural changes in the economy in the transformation period that resulted in scaling down or liquidation of industries which proved to be uncompetitive after the transition to the market economy. The problem of structural mismatch between the demand for labour and the supply of labour together with the questionable setting of the social system and hysteresis in the labour market resulted in particular in the rising long-term rate of unemployment which was reflected in high level of the NAIRU. However, its level begins to fall in the most recent period.

Further development of this issue is being considered in two directions. On one hand, it is possible to use a similar type of a model that is used by the Kalman filter, for the current estimation of the NAIRU and potential output. Another analytical option is the use of the VAR methodology, probably on the basis of its VEC model version. The model used in Chapter 3 can be always transformed into the VAR form.

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Appendix 1

Kalman filter

The purpose of the Appendix is to describe the functioning of the procedure of producing the estimate of a stochastic parameter by means of the Kalman filter. The description of the estimation process presented in this paper is based on Richardson (2000).

The Kalman filter has two stages: filtering and smoothing. Let us proceed from the generally expressed model which has been used in chapter 4. The first equation describes an observable variable and the second equation an unobservable variable:

$$\begin{aligned} Y_t &= ZX_t + RD_t + \nu_t \\ X_t &= TX_{t-1} + \eta_t \end{aligned},$$

where Z, R and T are matrices of coefficients, Y and X are vectors of observable or unobservable variables, D is the vector of exogeneous variables, while ν and η are stochastic elements with co-variation matrices H and Q:

$$\begin{aligned} H_t &= \sigma_\nu^2, \\ Q_t &= \sigma_\eta^2 r \end{aligned}.$$

hence, r is nothing else but „the signal-to-noise ratio“.

With the availability of new information on observable variables, through the filtering procedure estimates of unobservable variables are produced. Let us assume that A_t is an optimum estimate of vector X_t and P_t the relevant co-variation matrix. Then the optimum estimate when knowing A_{t-1} and P_{t-1} can be expressed as follows:

$$\begin{aligned} &A_{t+1/t} = (T - K_t Z)A_{t/t-1} + K_t(Y_t - D_t), \\ &where\ K_t = TP_{t/t-1}Z'F_t^{-1}\ a\ F_t = ZP_{t/t-1}Z + H_t\ . \\ &P_{t+1/t} = T\left(P_{t/t-1} - P_{t/t-1}Z'F_t^{-1}ZP_{t/t-1}\right)T' + Q_t \end{aligned}$$

On the basis of these estimates an estimate error is calculated:

$$\upsilon_t = Y_t - ZA_{t/t-1} - RD_t .$$

The estimate error calculated in this manner is part of the likelihood function that needs to be maximized. This is a criterion of an optimum estimate:

$$l = -\frac{1}{2}\log 2\pi - \frac{1}{2}\log|F_t| - \frac{1}{2}\upsilon_t F_t^{-1}\upsilon_t .$$

At the second stage, all the available information is used. This is a backward recursive computation that starts with the last estimate that has arisen from the filtering procedure (time T) and continues to the beginning of the used sample of data:

$$A_{t/T} = A_t + P_t^*\left(A_{t+1/T} - T_{t+1}A_t\right)$$
$$P_{t/T} = P_t + P_t^*\left(P_{t+1/T} - P_{t+1/t}\right)P_t^{*\prime} .$$
$$P_t^* = P_t T_{t+1}' P_{t+1/t}^{-1}$$

Annex 1

Table 2: Estimation of the quarterly NAIRU by means of the OECD method

quarter	3	4	1/98	2	3	4	1/99	2	3
NAIRU	6,34	6,53	6,72	6,90	7,08	7,25	7,41	7,56	7,70
quarter	4	1/00	2	3	4	1/01	2	3	4
NAIRU	7,82	7,92	8,00	8,06	8,11	8,15	8,17	8,18	8,18
quarter	1/02	2	3	4	1/03	2	3	4	1/04
NAIRU	8,17	8,16	8,14	8,12	8,10	8,06	8,03	7,99	7,93
quarter	2	3	4	1/05	2	3	4	1/06	2
NAIRU	7,88	7,82	7,74	7,66	7,57	7,47	7,36	7,25	7,13
quarter	3	4	1/07	2	3				
NAIRU	6,85	6,69	6,53	6,37	6,21				

Table 3: Estimation of the quarterly NAIRU by means of the Kalman filter

quarter	1/97	2	3	4	1/98	2	3	4	1/99
NAIRU	6,98	6,77	6,59	6,52	6,64	6,97	7,31	7,41	7,47
quarter	2	3	4	1/00	2	3	4	1/01	2
NAIRU	7,41	7,36	7,20	7,05	7,01	7,05	7,13	7,16	7,12
quarter	3	4	1/02	2	3	4	1/03	2	3
NAIRU	7,04	6,90	6,76	6,69	6,68	6,70	6,82	6,95	7,04
quarter	4	1/04	2	3	4	1/05	2	3	4
NAIRU	7,87	7,80	7,72	7,63	7,52	7,41	7,28	7,14	7,00
quarter	1/06	2	3	4	1/07	2	3		
NAIRU	6,12	5,99	5,91	5,85	5,78	5,82	5,95		

Annex 2

Table 4: **Sensitivity analysis (The sensitivity analysis for the quarterly data implies that the choice of the initial value has basically minimum effect on the end of the estimated series whereas the levels at the beginning of the estimated series are affected*)***

quarter	1/97	2	3	4	1/98	2	3	4	1/99
NAIRU (6 %)	*6,08*	*5,98*	*5,89*	*5,89*	*6,11*	*6,37*	*6,65*	*6,73*	*6,75*
quarter	2	3	4	1/00	2	3	4	1/01	2
NAIRU (6 %)	*6,67*	*6,58*	*6,42*	*6,27*	*6,21*	*6,22*	*6,31*	*6,35*	*6,33*
quarter	3	4	1/02	2	3	4	1/03	2	3
NAIRU (6 %)	*6,26*	*6,12*	*5,99*	*5,94*	*6,15*	*6,39*	*6,51*	*6,74*	*6,83*
quarter	4	1/04	2	3	4	1/05	2	3	4
NAIRU (6 %)	*6,84*	*6,81*	*6,72*	*6,59*	*6,50*	*6,39*	*6,25*	*6,17*	*6,13*
quarter	1/06	2	3	4	1/07	2	3		
NAIRU (6 %)	*6,05*	*5,92*	*5,85*	*5,78*	*5,73*	*5,80*	*5,94*		
quarter	1/97	2	3	4	1/98	2	3	4	1/99
NAIRU (7 %)	*6,98*	*6,77*	*6,59*	*6,52*	*6,64*	*6,97*	*7,31*	*7,41*	*7,47*
quarter	2	3	4	1/00	2	3	4	1/01	2
NAIRU (7 %)	*7,41*	*7,36*	*7,20*	*7,05*	*7,01*	*7,05*	*7,13*	*7,16*	*7,12*
quarter	3	4	1/02	2	3	4	1/03	2	3
NAIRU (7 %)	*7,04*	*6,90*	*6,76*	*6,69*	*6,68*	*6,70*	*6,82*	*6,95*	*7,04*
quarter	4	1/04	2	3	4	1/05	2	3	4
NAIRU (7 %)	*7,05*	*7,01*	*6,92*	*6,78*	*6,68*	*6,54*	*6,36*	*6,26*	*6,21*
quarter	1/06	2	3	4	1/07	2	3		
NAIRU (7 %)	*6,12*	*5,99*	*5,91*	*5,85*	*5,78*	*5,82*	*5,95*		
quarter	1/97	2	3	4	1/98	2	3	4	1/99
NAIRU (8 %)	*7,96*	*7,64*	*7,37*	*7,27*	*7,41*	*7,81*	*8,19*	*8,29*	*8,32*
quarter	2	3	4	1/00	2	3	4	1/01	2
NAIRU (8 %)	*8,20*	*8,11*	*7,90*	*7,72*	*7,69*	*7,73*	*7,79*	*7,79*	*7,71*
quarter	3	4	1/02	2	3	4	1/03	2	3
NAIRU (8 %)	*7,62*	*7,46*	*7,30*	*7,19*	*7,13*	*7,09*	*7,18*	*7,28*	*7,36*
quarter	4	1/04	2	3	4	1/05	2	3	4
NAIRU (8 %)	*7,35*	*7,29*	*7,20*	*7,04*	*6,92*	*6,75*	*6,52*	*6,37*	*6,29*
quarter	1/06	2	3	4	1/07	2	3		
NAIRU (8 %)	*6,19*	*6,05*	*5,98*	*5,92*	*5,82*	*5,82*	*5,94*		