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EXTENDED DSGE MODEL OF THE CZECH ECONOMY

ILKIN ALIYEV
BOŽENA BOBKOVÁ
ZBYNĚK ŠTORK

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Ilkin Aliyev
Ministry of Finance
Letenská 15
Prague 1, 118 10
Czech Republic
e-mail: ilkin.aliyev@mfcr.cz

Božena Bobková
Ministry of Finance
Letenská 15
Prague 1, 118 10
Czech Republic
e-mail: bozena.bobkova@mfcr.cz

Zbyněk Štork
Ministry of Finance
Letenská 15
Prague 1, 118 10
Czech Republic
e-mail: zbynek.stork@mfcr.cz

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Abstract

In the paper we present a New-Keynesian small open-economy dynamic stochastic general equilibrium model. It consists of four main building blocks: households, firms, government (fiscal and monetary authorities) and the foreign economy. The model is designed to suit the requirements of the Ministry of Finance of the Czech Republic on macroeconomic and fiscal policy analysis. The paper builds on previous versions of the model – a rather general first version and extension of fiscal policy block – and brings two important additional extensions. First, previously modelled net exports are split into two – exports and imports. Second, the model explicitly specifies investments as a separate part of the domestic demand. Therefore we included physical capital and capital services into the model. We believe that these extensions will be useful for in-depth analysis of various macroeconomic events as well as intended fiscal policy measures including their impacts on the Czech economy.

Abstrakt

Ve studii prezentujeme dynamický stochastický model všeobecné rovnováhy pro otevřenou ekonomiku Novokeynesiánské povahy. Skládá se ze čtyř hlavních bloků: domácností, firem, vlády (fiskální a monetární autority) a zahraniční ekonomiky. Model je nastavený pro potřeby makroekonomických a fiskálních analýz ministerstva financí ČR. Tato studie vychází z předchozích verzí tohoto modelu – spíše obecné první verze a rozšíření fiskálního bloku – a přináší dvě další rozšíření. Za prvé dříve modelované čisté exporty jsou nyní rozděleny na dvě části – export a import. Za druhé model explicitně specifikuje investice jako samostatnou část domácí poptávky. Z tohoto důvodu byl nově do modelu zahrnut i kapitál. Věříme, že tato rozšíření budou užitečná pro hloubku možných analýz různých makroekonomických událostí stejně jako zamýšlených fiskálních opatření a jejich dopadů do české ekonomiky.

Keywords: Dynamic stochastic general equilibrium model, fiscal policy, taxes, impulse response functions.

JEL classification: D58, E62, H30, H20.

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The views expressed in the paper do not necessarily reflect those of the Ministry of Finance of the Czech Republic. And of course, all mistakes are our own.

Contents

Non-technical summary

The model is New-Keynesian small open-economy dynamic stochastic general equilibrium model consisting of four main building blocks: households, firms, government (fiscal and monetary authorities) and the foreign economy. Important features of the model include heterogeneity among households (Saver and Spender type households), habit persistence in consumption, nominal rigidities (such as staggered prices and wages), real rigidities (adjustment costs in investments and the capital utilization rate) and stochastic technology trends in real sector variables.

Households are of two types: Savers and Spenders. Savers (or sometimes referred to as Ricardians) are able to smooth their consumption as they have an access to capital market.They own the firms and the physical capital, and they are also able to negotiate their wages. Spenders (or Rule of Thumbs), on the other hand, only derive their income from wages and social benefits, all of which is to be spent on consumption.

On the supply side, we assume a Cobb-Douglas production technology with two factors, labour and capital. Firms create demand for labour and negotiate wages with savers' households. In accordance with the New-Keynesian literature we introduce nominal and real rigidities. Sticky prices and wages are considered to be nominal rigidities. Here, we follow the methodology suggested by Calvo (1983) so that, during a specific period, only some firms are able to optimize prices. Those firms that are not able to do so would stay at the same price level as in the previous period. Similarly, only a fraction of households will be able to negotiate a new wage, others will work for the unchanged wages (Erceg et al., 2000). The variables utilization rate, investment and capital adjustment cost, have been adopted as real rigidities.

The government conducts monetary and fiscal policy. The first consists of a simple monetary policy rule that transmits movements in the output gap and inflation onto the short-term interest rate. The fiscal policy block contains tax income on the revenue side, outlays on social benefits and government consumption on the expenditure side.

When analyzing the Czech economy, which is extremely open, the foreign sector plays a crucial role in the model. We have used the Eurozone countries as a proxy for the foreign economy. Moreover, these countries include our main trading partners.

1 Introduction

Over the last decade, the New-Keynesian dynamic stochastic general equilibrium models (DSGE) have become very popular. Many central banks and other policy-making institutions have developed their DSGE models as a sophisticated tool for policy analysis and forecasting. The Swedish central bank-Sveriges Riksbank was the pioneering central bank in terms of employing DSGE (Adolfson et al., 2007b) model in their policy-making process. The DSGE models can be also found in the central banks of Norway - NEMO (Brubakk et al., 2006), Canada - ToTem (Murchison and Rennison, 2006) or US Federal Reserve - EDO (Chung et al., 2010) and others. Many policy-making institutions such as the International Monetary Fund - GIMF (Kumhof et al., 2010) or the QUEST of European Commission (Ratto et al., 2009) implemented these models as well.

The Ministry of Finance of the Czech republic has developed an extended version of the DSGE model. It serves various purposes. It primarily supports macroeconomic forecasts by evaluating model scenarios on a quarterly basis. Moreover, the model is employed for simulation purposes related to changes in fiscal policy parameters, and also for assessing the sensitivity of macroeconomic variables to various shocks to the economy.

The initial version of the model (Štork et al., 2009) introduced rather simple general equilibrium framework for the Czech economy with standard features of New-Keynesian economics. Subsequently, the fiscal part of the model was further extended in Štork and Závacká (2010) to facilitate the production of simulations for different fiscal policy variables. The current major extension of the model builds on the previous versions and develops them in two main areas. First, previously modelled net exports are split into two - exports and imports. Second, the model explicitly specifies investments that used to be a part of domestic demand together with private consumption. This also requires the inclusion of physical capital and capital services. The latter is quite an extensive modification as it affects all parts of the model. Therefore, in the paper, we decided to present the model as a whole rather than as an update. We believe that the reader might benefit from that.

In the Czech Republic, the dynamic stochastic general equilibrium model is also used for the forecasting and simulation purposes in the Czech National Bank (CNB) - the G3 model (Andrle et al., 2009). The main difference between these two models is based on the agenda of institutions where they are applied. While the model of central bank targets and evolves in detail especially inflation and monetary policy, our model expands taxes and fiscal policy. Moreover, other more technical differences between these two models can be found, for instance CNB works with the nonstationary model, while our model is implicitly stationarized. The central bank employs forward looking expections and we assume heterogenous households.

The remainder of the paper is organized as follows. Chapter 2 introduces all parts of the model including the main equations. Chapter 3 specifies steady states, the derived log-linearized system and touches upon the data used in our analysis. It also contains the calibration and estimation of the model parameters. Chapter 4 describes the results from various simulation exercises. Chapter 5 is the conclusion.

2 Structure of the model

2.1 Notation in the paper

For a better understanding of the notations made throughout the paper, we will outline some basic principles from the outset. The variables can appear in the following forms:

- X_t levels (number of persons, financial values in CZK, price indices);
- x_t detrended values of X_t using stochastic productivity trend z_t. i.e. $x_t = X_t/z_t$;
- $\bar{X}_t$ steady state of variable with trend z_t;
- $\bar{x}$ steady state of detrended variable x_t;
- $\bar{X}$ steady state of stationary variable without trend z_t. i.e. price index is not subject to a trend z_t;
- $\hat{x}_t$ deviation from steady state of detrended variable x_t, such as that $x_t = \bar{x}(1 + \hat{x}_t)$;
- $\hat{X}_t$ deviation from steady state of stationary variable without trend z_t, such as that $X_t = \bar{X}_t(1 + \hat{X}_t)$.

2.2 Households

We assume heterogenous households. Following Mankiw (2000), households are divided into two groups: *Savers* and *Spenders*. Traditional dynamic general equilibrium models rely on representative households that maximize utility stemming from consumption and leisure. These households own physical capital which they rent to firms, and they also own financial capital. At each period they make decisions on consumption, investment, leisure and asset holdings in such a way as to maximize their life-time utility. At the same time, restrictions on the form of utility function lead to consumption smoothing decision-making. These types of households are referred to as *Savers*.

The ability to save allows Savers to distribute the income shock into future consumption by not accommodating it entirely in the current period's consumption. However, the empirical literature argues that such highly consumption smoothing behavior is not what happens in reality. Data show a much higher correlation between current income and current consumption. Mankiw (2000) argues that a significant part of households does not (and/or cannot) smooth and consume their entire income. These households are called *Spenders*. Sometimes they are referred to as "rule-of-thumb" or "non-Ricardian" households. The inclusion of Spenders into model increases the sensitivity of current consumption to the income shock which leads to a better alignment with empirical observations. Furthermore, Gali et al. (2007) argue that classic real business cycle models are not able to explain the positive response of consumption to government expenditure increase apparent in empirical evidence. Hence, they also suggest the inclusion of the "rule-of-thumb" households into the standard New-Keynesian framework.

Habit formation or habit persistence in consumption, as developed in Abel (1990) and supported by Fuhrer (2000) is a commonly used feature in the recent structural macroeconomic modeling literature (Smets and Wouters, 2003; Christiano et al., 2005, etc.). Under habit persistence, marginal utility of consumption in the current period will be further lowered with the increase of current consumption, whereas marginal utility in the next period will be increased. As such it provides extra force for household consumption smoothing (Lettau and Uhlig, 2000). Constantinides (1990) argues for inclusion of habit formation in consumption as it can provide a reasonable solution for the equity premium puzzle identified by Mehra and Prescott (1985). In classical models, equity premium puzzle arises along with consumption response to high interest spreads of risky assets, unless extremely high risk aversion of households is assumed. The required low volatility in consumption can be achieved with habit formation.

Moreover, the habit formation assumption entails important consequences for real business cycles. Modern business cycle models contain various nominal and real rigidities, and other kinds of inefficiencies, in order to capture the behavior and the response of the main macroeconomic variables. In these models, inclusion of habit formation plays a central role in capturing consumption response to shocks. The reaction of consumption to various shocks is not contemporaneous. It is shaped more like a hump, reaching its peak over time, with the impact tailing off in later periods. By assuming habit persistence one could provoke a response of this nature.

2.2.1 Savers

In our model, only Savers have habit persistence. The benefit of a framework that assumes heterogeneity of households and habit formation is that it can give a hump-shaped response of consumption without hampering the positive sign of the response of consumption to positive income shocks. To achieve cyclical responses consistent with the data, habit formation along with the various kinds of nominal and real rigidities became a standard features of the current New-Keynesian models.

We assume a continuum of infinitely lived households, indexed $j \in [0, 1]$, of which a fraction α_R are Savers type and $1 - \alpha_R$ are Spenders type. In our model, Saver households contain all the classic features relevant to standard New-Keynesian models. A representative Saver household's budget constraint has the following form:

$$\begin{aligned}(1+\tau_t^c)P_t^C C_{j,t}^R + P_t^I I_{j,t} + P_t a(u_{j,t})K_{j,t}^s + \tfrac{1}{R_t}B_{j,t+1} + \tfrac{1}{R_t^*+\zeta_t}S_t B_{j,t+1}^* = \\ = B_{j,t} + S_t B_{j,t}^* + (1-\tau_t^k)[R_t^k u_{j,t} K_{j,t}^s + Q_t] + (1-\tau_t^w)W_t N_{j,t}^R\end{aligned} \quad (1)$$

The households earn a net wage income $(1-\tau_t^w)W_tN_{j,t}^R$ by supplying labour $N_{j,t}^R$. They also receive profits Q_t from ownership of firms and capital rents $R_t^k u_{j,t}K_{j,t}^s$, which are both taxed at the same corporate tax rate τ_t^k. Here, R_t^k is the rental rate of the capital and $u_{j,t}$ is utilization rate of the capital stock $K_{j,t}^s$ with $u_{j,t} = K_{j,t}/K_{j,t}^s$. $K_{j,t}$ is the part of the capital stock that is actually rented to firms and enters into the production function/process. The households also

Along with expenditures on consumption $C_{j,t}^R$, Savers incur expenditures on investment $I_{j,t}$ (with investment price P_t^I) contributing to the accumulation of physical capital. Capital stock evolves with depreciation rate δ as:

$$K_{j,t+1}^s = (1-\delta)\,K_{j,t}^s + F\left(I_{j,t}, I_{j,t-1}\right) \tag{2}$$

Following Christiano et al. (2005), we assume that the function F summarizes the investment technology transforming current and past investments into physical capital used in the following period. It represents investment adjustment costs and has the following form:

$$F\left(I_t, I_{t-1}\right) = \left[1-\Upsilon\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{3}$$

with the function Υ having these properties at steady-state $\Upsilon(ss) = 0$, $\Upsilon'(ss) = 0$, and $\kappa \equiv \Upsilon''(ss) > 0$. As argued in Christiano et al. (2005), it is sufficient for the solution procedure to know the value of the adjustment cost parameter κ;[1] and no other features of Υ need to be specified. We will further discuss the properties of Υ and assume its exact functional form for a clearer presentation later. As households own the entire physical capital, they also pay the cost of setting the utilization (or capital adjustment costs) $a(u_t)K_t^s$ in units of consumption good, $a(u_t)$ being the increasing convex function.

Saver households hold financial assets in the form of domestic government bonds $B_{j,t}$ and foreign bonds $B_{j,t}^*$ with the interest rates R_t and R_t^*, respectively. The households also receive risk premium ζ_t while holding the foreign bonds.[2] Both interest rates are gross rates. During each period, these households decide how much of either domestic or foreign assets to hold. We assume that there is infinite demand for domestic bonds. Whenever the government issues new debt, it creates sufficient demand for its bonds to ensure it can sell any amount at any price. The amount of foreign assets held by households is determined as the residual from the budget constraint.

In each period t, a representative Saver household chooses paths for $C_{j,t}^R$, $I_{j,t}$, $N_{j,t}^R$, $u_{j,t}$, $K_{j,t+1}^s$, $B_{j,t+1}$ and $B_{j,t+1}^*$ to maximize the expected infinite horizon utility as below

[1] Explicit form of the investment adjustment cost function can be found in Section 2.2.4

[2] The risk premium is a function of net foreign assets, NFA/GDP more specifically. From a balance of payments we know that $\Delta NFA_t \approx CA_t + FA_t$, where CA_t stands for the current account and FA_t the financial account. Since many transactions from the financial account are sometimes sterilized, we decided to ignore this component of net financial assets in the model.

subject to constraints (1) and (2) given P_t^C, P_t^I, R_t, R_t^*, W_t, S_t, τ_t^c, τ_t^k, τ_t^w and function F as in (3):

$$E_t \sum_{n=0}^{\infty} \beta^n U_{j,t+n} = E_t \sum_{n=0}^{\infty} \beta^n \left[\log \left(C_{j,t+n}^R - H_{j,t+n} \right) - \frac{(N_{j,t+n}^R)^{1+\psi_N}}{1+\psi_N} \right] \tag{4}$$

where, β is the discount factor, $H_t = h_R C_{t-1}$ stands for the external habit formation, $\psi_N > 0$ - is the inverse of the real wage elasticity of labour supply.

Assigning λ_t and ϕ_t to be Lagrangian multipliers for the constraints (1) and (2), respectively, maximization of the Lagrangian yields the following first-order conditions:

$$C_{j,t}^R \Rightarrow \left(C_{j,t}^R - h_R C_{j,t-1}^R \right)^{-1} = \lambda_t (1+\tau_t^c) P_t^C \tag{5}$$

$$\begin{aligned} I_{j,t} \Rightarrow \lambda_t P_t^I = \phi_t \left[1 - \Upsilon \left(\tfrac{I_{j,t}}{I_{j,t-1}} \right) - \Upsilon' \left(\tfrac{I_{j,t}}{I_{j,t-1}} \right) \tfrac{I_{j,t}}{I_{j,t-1}} \right] + \\ + \beta E_t \left[\phi_{t+1} \Upsilon' \left(\tfrac{I_{j,t+1}}{I_{j,t}} \right) \left(\tfrac{I_{j,t+1}}{I_{j,t}} \right)^2 \right] \end{aligned} \tag{6}$$

$$K_{j,t+1}^s \Rightarrow \phi_t = E_t \{ \beta (1-\delta) \phi_{t+1} + \beta \lambda_{t+1} \left[\left(1 - \tau_{t+1}^k \right) R_{t+1}^k u_{j,t+1} - P_{t+1} a(u_{j,t+1}) \right] \} \tag{7}$$

$$u_{j,t} \Rightarrow P_t a'(u_{j,t}) = \left(1 - \tau_t^k \right) R_t^k \tag{8}$$

$$B_{j,t+1} \Rightarrow \lambda_t = \beta R_t E_t \left(\lambda_{t+1} \right) \tag{9}$$

$$B_{j,t+1}^* \Rightarrow \lambda_t S_t = \beta (R_t^* + \zeta_t) E_t \left(\lambda_{t+1} S_{t+1} \right) \tag{10}$$

$$N_{j,t}^R \Rightarrow \left(N_{j,t}^R \right)^{\psi_N} = \lambda_t \left(1 - \tau_t^w \right) W_t \tag{11}$$

2.2.2 Spenders

On the other hand, Spender households only collect income from wages and social benefits, and spend their entire income on consumption. In other words, they decide neither on expenditure level nor on their income levels, and thus cannot establish any habit persistence in consumption. Also, being wage-takers they have inelastic labour supply, and they do not have access to financial markets. We assign a budget constraint for a representative Spender household to be:

$$(1+\tau_t^c) P_t^C C_{j,t}^N = (1-\tau_t^w) W_t N_{j,t}^N + \tau_t^b W_b (N_{j,t}^N - L_{j,t}^N) + TR_t \tag{12}$$

where $C_{j,t}^N$ and $N_{j,t}^N$ are real consumption expenditures and labour supply by the Spender household, respectively, P_t^C denotes the price of the consumption good, $L_{j,t}^N$ stands for the labor demand, W_t is the wage, W_b stands for the wage bill and τ_t^c is the consumption tax rate. Also, they pay payroll tax τ_t^w, and receive government transfers TR_t and unemployment benefits τ_t^b based on unemployment.

The variables that the model is built on, such as consumption, investment, income, etc., are not stationary as they grow over time. We assume that there is a permanent technology shock z_t causing real variables to grow on average. Following Adolfson et al. (2007a), the stochastic technology trend is given by:

$$\mu_{z,t} = \frac{z_t}{z_{t-1}} \tag{13}$$

and

$$\mu_{z,t} = (1 - \rho_{\mu_z})\,\mu_z + \rho_{\mu_z}\mu_{z,t-1} + \epsilon_{z,t} \tag{14}$$

where, μ_z is the steady-state growth rate of technology, and ρ_{μ_z} is the autoregressive parameter.

2.2.3 Consumption

From the first-order condition for consumption (5), we see that consumption variable $C_{j,t}^R$ is the only one indexed with j. Therefore, the consumption choice will be the same for each household, $C_t^R \equiv C_{j,t}^R$. In an attempt to make consumption stationary we detrend it by technology z_t. Denoting $c_t^R \equiv \frac{C_t^R}{z_t}$ we adjust first-order condition for Saver's consumption (5) as:

$$\left(c_t^R - h_R c_{t-1}^R \frac{1}{\mu_{z,t}}\right)^{-1} = \lambda_t z_t (1 + \tau_t^c) P_t^C \tag{15}$$

As Spenders consume entire labour income in each period, their consumption level is obtained from their budget constraint. Given that Spender households are homogenous, then we can write $N_t^N \equiv N_{j,t}^N$ and $C_t^N \equiv C_{j,t}^N$. With $c_t^N \equiv \frac{C_t^N}{z_t}$ in stationary form it becomes:

$$(1 + \tau_t^c) P_t^C c_t^N = \frac{1}{z_t}\{(1 - \tau_t^w) W_t N_{j,t}^N + \tau_t^b W_b (N_{j,t}^N - L_{j,t}^N) + TR_t\} \tag{16}$$

Assuming that the economy consists of an infinite number of households indexed between [0,1] of which a fraction α_R are of Saver type, then aggregate stationarized consumption of households, denoted as $c_t \equiv \frac{C_t}{z_t}$, is given by:

$$c_t = \alpha_R c_t^R + (1 - \alpha_R)\, c_t^N \tag{17}$$

In an open economy settings, such as the Czech economy , we assume that consumption goods can be produced domestically (C_t^d) or imported (C_t^m). Aggregate consumption is given as CES aggregation of both type of consumption goods:

$$C_t = \left[(1 - \mu_{cm})^{1-\theta_c} \left(C_t^d\right)^{\theta_c} + (\mu_{cm})^{1-\theta_c} \left(C_t^m\right)^{\theta_c}\right]^{\frac{1}{\theta_c}} \tag{18}$$

where μ_{cm} is the share of imports in consumption, and $1/(1-\theta_c)$ is the elasticity of substitution between both consumption goods ($\theta_c < 1$). After choosing an intertemporal consumption level, households make intratemporaral optimizations between domestic and imported consumption goods during each period. Here, given prices for domestic (P_t^d) and imported (P_t^m) consumption goods, they minimize nominal consumption expenditures. In other words, they choose C_t^d and C_t^m to minimize $P_t^d C_t^d + P_t^m C_t^m \equiv P_t^C C_t$ subject to (18) given prices. Such an optimization yields the following demand functions:

$$c_t^d = (1-\mu_{cm}) \left(\frac{P_t^d}{P_t^C} \right)^{\frac{1}{\theta_c - 1}} c_t \tag{19}$$

$$c_t^m = \mu_{cm} \left(\frac{P_t^m}{P_t^C} \right)^{\frac{1}{\theta_c - 1}} c_t \tag{20}$$

with $c_t^d \equiv \frac{C_t^d}{z_t}$, $c_t^m \equiv \frac{C_t^m}{z_t}$ and consumption price index P_t^C given by:

$$P_t^C = \left[(1-\mu_{cm}) \left(P_t^d \right)^{\frac{\theta_c}{\theta_c - 1}} + (\mu_{cm}) \left(P_t^m \right)^{\frac{\theta_c}{\theta_c - 1}} \right]^{\frac{\theta_c - 1}{\theta_c}} \tag{21}$$

2.2.4 Investment and capital

Investments in physical capital are also the same across all Savers. We stationarize investment using $i_t \equiv \frac{I_t}{z_t}$ and obtain the following equation from the first-order condition for investments as in (7):

$$\begin{aligned} \lambda_t P_t^I = \phi_t \left[1 - \Upsilon \left(\tfrac{i_t}{i_{t-1}} \mu_{z,t} \right) - \Upsilon' \left(\tfrac{i_t}{i_{t-1}} \mu_{z,t} \right) \tfrac{i_t}{i_{t-1}} \mu_{z,t} \right] + \\ + \beta E_t \left[\phi_{t+1} \Upsilon' \left(\tfrac{i_{t+1}}{i_t} \mu_{z,t} \right) \left(\tfrac{i_{t+1}}{i_t} \mu_{z,t+1} \right)^2 \right] \end{aligned} \tag{22}$$

If we apply log-linearization around steady-state to the equation above, we do not need to assume any functional form for Υ. However, for the sake of clarity we can assume $\Upsilon \left(\frac{I_t}{I_{t-1}} \right) = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - \mu_z \right)^2$ that satisfies $\Upsilon(\mu_z) = 0$, $\Upsilon'(\mu_z) = 0$, and $\kappa \equiv \Upsilon''(\mu_z) > 0$ at steady-state. Such functional form charges penalty for sharp changes in investments and reduces the reflection of highly volatile investments on capital stock.

For the law of motion of capital stock we use past technology level for detrending $k_t^s \equiv \frac{K_t^s}{z_{t-1}}$, because capital stock is a predetermined variable. The capital stock for the next period will automatically be determined every time after an investment decision is made.

$$k_{t+1}^s = (1-\delta) \frac{k_t^s}{\mu_{z,t}} + \left[1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} \mu_{z,t} - \mu_z \right)^2 \right] i_t \tag{23}$$

On the other hand, capital services demanded by firms K_t will be stationarized with the current technology level $k_t \equiv \frac{K_t}{z_t}$. For the utilization rate of capital this implies:

$$u_t = \frac{k_t}{k_t^s}\mu_{z,t} \tag{24}$$

As with consumption, total investments is a CES aggregate of domestic I_t^d and imported I_t^m investment goods:

$$I_t = \left[(1-\mu_{im})^{1-\theta_i}\left(I_t^d\right)^{\theta_i} + (\mu_{im})^{1-\theta_i}\left(I_t^m\right)^{\theta_i}\right]^{\frac{1}{\theta_i}}$$

where μ_{im} is the share of imports in investments, and $1/(1-\theta_i)$ is the elasticity of substitution between both investment goods ($\theta_i < 1$). Households also make intratemporal investment decisions, choosing between domestic and imported investment goods through the minimization of nominal investment expenditures $P_t^d I_t^d + P_t^m I_t^m \equiv P_t^I I_t$. This yields the following stationarized individual investment demand equations with $i_t^d \equiv \frac{I_t^d}{z_t}$ and $i_t^m \equiv \frac{I_t^m}{z_t}$:

$$i_t^d = (1-\mu_{im})\left(\frac{P_t^d}{P_t^I}\right)^{\frac{1}{\theta_i-1}} i_t \tag{25}$$

$$i_t^m = \mu_{im}\left(\frac{P_t^m}{P_t^I}\right)^{\frac{1}{\theta_i-1}} i_t \tag{26}$$

Similarly, the aggregate investment price is given by:

$$P_t^I = \left[(1-\mu_{im})\left(P_t^d\right)^{\frac{\theta_i}{\theta_i-1}} + (\mu_{im})\left(P_t^m\right)^{\frac{\theta_i}{\theta_i-1}}\right]^{\frac{\theta_i-1}{\theta_i}} \tag{27}$$

2.2.5 Solution of the first order conditions

Sometimes it is useful to get rid of bit annoying Lagrange multipliers in the first order condition equations. We use the first order condition for bonds $\beta R_t = \frac{\lambda_t}{\lambda_{t+1}}$ to eliminate λ from the other conditions. According to Senecca, 2010, Stahler and Thomas, 2011,Vukotic, 2007, we can define Tobin's Q TQ_t as the fraction of the two Langrange multipliers $\frac{\phi_t}{\lambda_t}$. Further, we can apply this Tobin's Q in the investment and capital equations to eliminate ϕ. This approach simplifies the solution of the model and makes results easier to understand.

2.3 Firms

2.3.1 Producers

There is continuum of intermediate goods producing firms of measure 1, firm i at time t produces intermediate good Y_{it}. There is also a final goods producing firm with output Y_t. Each intermediate good producer is a monopoly supplier of that good in a competitive market for inputs. The final goods producing firm transforms intermediate goods into homogenous goods using CES production technology:

$$Y_t = \left[\int_0^1 Y_{it}^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{28}$$

with $\theta > 1$. The final good is then used for consumption and investments. The final goods firm maximizes its profits by choosing the amount of different inputs given output price P_t and all input prices P_{it}.

$$\max_{Y_{it}} P_t Y_t - \int_0^1 P_{it} Y_{it} di$$

Optimization leads to the demand for intermediate good i as follows:

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\theta} Y_t \tag{29}$$

Equations (28) and (29) together help us to obtain the price of the final good in relationship with the prices of the intermediate goods:

$$P_t = \left[\int_0^1 P_{it}^{1-\theta} di \right]^{1/(1-\theta)}$$

The output of an intermediate good producer is given by the following production technology:

$$Y_{it} = z_t^{1-\eta} K_{it}^{\eta} L_{it}^{1-\eta}$$

where, $\eta < 1$, K_{it} is rented capital services and L_{it} is hired labour by intermediate firm i.

At first, in each period an intermediate goods producer minimizes the cost of production $W_t L_{it} + R_t^k K_{it}$ subject to the production technology, given P_{it}, wages W_t and the rental rate of capital R_t^k. The Lagrangian for this problem is given as:

$$\min_{K_{it}, L_{it}} Ł = W_t L_{it} + R_t^k K_{it} + \nu_t \left[Y_{it} - z_t^{1-\eta} K_{it}^{\eta} L_{it}^{1-\eta} \right]$$

where, ν_t is the Lagrange multiplier. The first order conditions with respect to capital and labour are:

$$W_t = (1-\eta) \nu_t z_t^{1-\eta} K_{it}^{\eta} L_{it}^{-\eta} \tag{30}$$

$$R_t^k = \eta \nu_t z_t^{1-\eta} K_{it}^{\eta-1} L_{it}^{1-\eta} \tag{31}$$

Using both the first-order conditions and the Cobb-Douglas production technology referred to above, we obtain a marginal cost marginal cost equation that will be the same for all intermediate goods:

$$MC_t = \left(\frac{1}{1-\eta} \right)^{1-\eta} \left(\frac{1}{\eta} \right)^{\eta} \frac{W_t^{1-\eta} R_t^{k\eta}}{z_t^{1-\eta}} \tag{32}$$

2.3.2 Price-setting

During each period, only a $1 - \xi_p$ $(0 < \xi_p < 1)$ fraction of intermediate good firms is able to set/adjust their prices, according to Calvo (1983) price stickiness. The pricing decision is based on the following maximization of discounted future profits:

$$\max_{P_{it}^{new}} E_t \sum_{n=0}^{\infty} (\beta \xi_p)^n \left[P_{it}^{new} Y_{i,t+n} - MC_{t+n} Y_{i,t+n} \right]$$

where P_{it}^{new} is a new price set by the firm. Substituting individual demand and marginal cost functions we obtain the following:

$$\max_{P_{it}^{new}} E_t \sum_{n=0}^{\infty} (\beta \xi_p)^n \left[P_{it}^{new} - \left(\frac{1}{1-\eta} \right)^{1-\eta} \left(\frac{1}{\eta} \right)^{\eta} \frac{W_{t+n}^{1-\eta} R_{t+n}^{k\ \ \eta}}{z_{t+n}^{1-\eta}} \right] \left(\frac{P_{it}^{new}}{P_{t+n}} \right)^{-\theta} Y_{t+n}$$

After simple manipulations with the first-order condition of the above optimization the following price-setting rule is obtained:

$$P_{it}^{new} = \frac{\theta}{\theta - 1} \frac{\sum_{n=0}^{\infty} (\beta \xi_p)^n Y_{t+n} P_{t+n}^{\theta} MC_{t+n}}{\sum_{n=0}^{\infty} (\beta \xi_p)^n Y_{t+n} P_{t+n}^{\theta}} \tag{33}$$

It can be seen that new price setting equation (33) does not include i on the right hand side. It follows that each firm that can set a price will choose the same price. Consequently, we can drop the index i denoting $P_{it}^{new} = P_t^{new}$. This leads to the fact that production volumes for each price-setter will be the same. So, we can write $Y_{it} = \tilde{Y}_t$. Also, we assume that those who cannot optimize will choose the same price level as the previous period. Therefore, the aggregate price index can be expressed as:

$$P_t^{1-\theta} = \xi_p P_{t-1}^{1-\theta} + (1 - \xi_p) \left(P_t^{new} \right)^{1-\theta} \tag{34}$$

2.4 Labour market

Each household j is a monopoly supplier of "household-specific" differentiated labour N_{jt} to "the labour bundler". The labour bundler anticipates the labour demand by firms L_t and has the following aggregation technology for the individual labours:

$$E_t(L_t) = \left[\int_0^1 N_{jt}^{(\theta_W - 1)/\theta_W} dj \right]^{\theta_W/(\theta_W - 1)}$$

where, $\theta_W > 1$. The labour bundling firm is perfectly competitive leading to the aggregate wage W_t equal to:

$$W_t = \left[\int_0^1 W_{jt}^{1-\theta_W} dj \right]^{1/(1-\theta_W)} \tag{35}$$

The demand for household's labour is given by:

$$N_{jt} = L_t \left(\frac{W_t}{W_{jt}} \right)^{\theta_W} \tag{36}$$

To incorporate wage staggering into the model, we follow the similar methodology to the one suggested by Erceg et al. (2000). As Savers have alternative sources of income, we assume that they can smooth their consumption, hence putting them in a negotiable position. Only a $1 - \xi_w$ $(0 < \xi_w < 1)$ fraction of Saver households can set their wages at time t. Households which can optimize their wage will choose the same wage $W_{jt} = W_t^{new}$ under the assumption of lump-sum insurance plan. Spenders cannot change their wages, and therefore their wage will be the same as in the previous period. It follows that the total number of households that will work for the same wage as in the previous period consists of all Spenders, $1 - \alpha_R$, and a ξ_w fraction of Savers, $\xi_w \alpha_R$. The sum of these two is equal to $1 - \alpha_R(1 - \xi_w)$. Consequently, the number of households who can optimize their wages equal to $\alpha_R(1 - \xi_w)$. Then, the wage aggregation given in (35) can be written as follows:

$$(W_t)^{1-\theta_W} = \alpha_R \left[\xi_w \left(W_{t-1} \right)^{1-\theta_W} + (1 - \xi_w) \left(W_t^{new} \right)^{1-\theta_W} \right] + (1 - \alpha_R) \left(W_{t-1} \right)^{1-\theta_W} \tag{37}$$

Households that can set their wages W_t^{new} maximize the utility function (4) subject to constraints (1) and (2) by choosing optimal wages after plugging in the individual labour demand function given by (36). With some manipulations, the first-order condition for such optimization becomes:

$$W_t^{new} = \frac{\theta_W}{\theta_W - 1} \frac{\sum_{n=0}^{\infty} (\beta \xi_w)^n \left(N_{jt+n} \right)^{1+\psi_N}}{\sum_{n=0}^{\infty} (\beta \xi_w)^n \left(1 - \tau_{t+n}^w \right) \lambda_{t+n} N_{jt+n}} \tag{38}$$

It follows from equation (36) that demand for wage optimizing Saver household's labour becomes:

$$N_{jt} = L_t \left(\frac{W_t}{W_t^{new}} \right)^{\theta_W} \tag{39}$$

2.5 Foreign sector

For an open economy, foreign sector variables have a crucial impact on economic dynamics. In our model, the domestic economy is linked to the foreign sector through trade, prices and financial markets.

Imports are defined as the sum of imported consumption C_t^m and imported investment I_t^m goods:

$$M_t = C_t^m + I_t^m \tag{40}$$

Similarly, exports consist of consumption C_t^{m*} and investment I_t^{m*} goods demanded by the foreign sector.

$$X_t = C_t^{m*} + I_t^{m*} \tag{41}$$

where C_t^{m*} and I_t^{m*} are assumed to behave in a similar way (in a foreign economy) as derived consumption and investment for the domestic economy introduced in Sections 2.2.3 and 2.2.4.

We assume that the law of one price still holds, meaning that foreign prices P_t^* and imported good prices P_t^m are linked through the nominal exchange rate S_t:

$$P_t^m = S_t P_t^* \tag{42}$$

The price of domestic good P_t is set by domestic firms with price stickiness a la Calvo (see the Firms Section).

Foreign interest rate affects the domestic economy through parity equation. From the first-order conditions for B_{t+1} and B_{t+1}^* we obtain an uncovered interest rate parity equation:

$$\frac{E_t\left(S_{t+1}\right)}{S_t} = \frac{R_t}{\left(R_t^* + \zeta_t\right)} \tag{43}$$

Due to small size of the domestic economy, all foreign variables are considered to be exogenous. In order to simulate and forecast the domestic economy, we need to estimate how the foreign sector will evolve. We assume that foreign variables follow simple AR(1) process. Denoting the foreign vector $F_t = [C_t^{m*}, I_t^{m*}, P_t^*, R_t^*]'$, the process can be represented as:

$$F_t = \Phi^* F_{t-1} + \epsilon_t^* \tag{44}$$

where, Φ^* is a diagonal matrix of autoregressive parameters $\begin{pmatrix} \phi_C^* & 0 & 0 & 0 \\ 0 & \phi_I^* & 0 & 0 \\ 0 & 0 & \phi_P^* & 0 \\ 0 & 0 & 0 & \phi_R^* \end{pmatrix}$.

2.6 Government

2.6.1 Fiscal authority

Fiscal policy is an important part of the model as it allows for simulation of various fiscal policy measures. However, the specification of the government sector will be kept quite simple using aggregated equations of revenues and expenditures.

Total government revenues (GR_t) consist of tax revenues. There are different taxes in the economy all paid by households: tax rate on consumption (τ_t^c), labour income tax rate (τ_t^w), and the corporate income tax rate (τ_t^k).

$$GR_t = \tau_t^c \left(P_t^C C_t + G_t\right) + \tau_t^w W_t L_t + \tau_t^k \left(R_t^k K_t + Q_t\right) \tag{45}$$

Following Forni et al. (2007), implicit tax rates are defined as a share of revenues from each tax collection at the respective tax base. Formally, we use the following equations to derive implicit tax rates for consumption (τ_t^c), wages (τ_t^w), corporate income tax (τ_t^k) and the rate for social benefits on the expenditure side (τ_t^b).

$$\tau_t^c = \frac{T_t^c}{P_t C_t}, \quad \tau_t^w = \frac{T_t^w}{W_t L_t}, \quad \tau_t^k = \frac{T_t^k}{GOS_t}, \quad \tau_t^b = \frac{T_t^b}{W_t N_t},$$

where:

T_t^c	budgetary income from taxes on consumption,
T_t^w	budgetary income from taxes on wages,
T_t^k	budgetary income from corporate taxes,
T_t^b	unemployment benefits expenditures,
GOS_t	gross operating surplus.

Implicit tax rates follow AR(1) process in a log-linear deviation from steady-state:

$$\hat{\tau}_t = \Gamma \hat{\tau}_{t-1} + \epsilon_t \tag{47}$$

where, $\hat{\tau}_t = [\hat{\tau}_t^c, \hat{\tau}_t^w, \hat{\tau}_t^k, \hat{\tau}_t^b]'$, and Γ is a diagonal matrix of AR(1) coefficients.

Government expenditures (GE_t) consist of consumption expenditures (G_t^C), unemployment benefits, other transfers to households (TR_t) other expenditures (G_t^O). They are largely exogenous, except government transfers to Spender households that are proportional to their labour income.

$$GE_t = G_t^C + \tau_t^b \bar{W}(N_t^N - L_t^N) + TR_t + G_t^O \tag{48}$$

The difference between government expenditures and revenues is financed through borrowing. In other words, the primary budget deficit $PD_t \equiv GE_t - GR_t$ must be equal to new debt $\frac{1}{R_t} B_{t+1} - B_t$.

$$GE_t - GR_t = \frac{1}{R_t} B_{t+1} - B_t \tag{49}$$

The parameters of AR (1) processes in the case of variables G_t^C and G_t^O are set to ensure the convergence and long-term stability of the model.

2.6.2 Monetary authority

We keep using quite standard monetary policy rule, developed by Taylor (1993) and further discussed by Svensson (1998). In log-linearized form, it is represented as:

$$R_t = (1 - \phi_r)[\bar{R} + \lambda_\pi \hat{\pi}_t + \lambda_y \hat{y}_t] + \phi_r R_{t-1} \tag{50}$$

where

R_t	is short-term nominal interest rate,
$\bar{R}$	steady state value of short-term interest rate,
$\hat{\pi}_t$	deviation of inflation rate from its steady state (target) value,
$\hat{y}_t$	output gap,
λ_y	output gap weight,
λ_p	inflation weight,
ϕ_r	interest rate smoothing parameter.

2.7 Aggregation and market clearing

The real economy must be in balance, so GDP identity must hold:

$$Y_t = C_t + \frac{G_t^C}{P_t} + I_t + X_t - M_t. \tag{51}$$

Total profits are the difference between production and costs of labour and capital:

$$Q_t = P_t Y_t - W_t L_t - R_t^k K_t. \tag{52}$$

3 Solution of the model

3.1 Data

We employ the quarterly data for the Czech economy for the calibration and estimation of the model parameters. Mostly, the time series of model variables are available starting from the year 1995, however, we use data after 2000, since the previous period data are affected by structural breaks and volatility. For the purposes of forecasting and scenario simulations, we will feed the model with quarterly data through endogenous state variables and exogenous variables.

Most real variables in the data, such as consumption, investments and GDP, among others, follow long-term trends. As discussed in the model section, we define the stochastic technology trend z_t that causes these variables to grow over time. We have selected the labour productivity trend as the stochastic technology trend, and detrend real variables with it. Formally, we compute it as $z_t = Y_t/L_t$, and assume the exogeneity of this type of technology trend. All components of GDP are stationarized.

Consumption data for Saver and Spender households can be obtained from the Household Budget Survey that is carried out on a quarterly basis by the Czech Statistical Office. As a representative for Saver households'consumption, C_t^R, we choose average consumption of households in the highest 30% income percentile. Similarly, for Spender households' consumption C_t^N we refer to the average consumption of the lowest 30% income percentile households. One way of finding aggregate consumption, C_t is to add both Savers' and Spenders' consumptions. However, the dynamics of this aggregate consumption may differ from the consumption in the National Accounts. In order to be consistent with aggregate data from the main GDP identity, we give preference to the aggregate consumption from the National Accounts. By taking Spenders' consumption from the households survey, we compute Savers' consumption as residual. This is is based on the assumption that high-income households have less incentive to report accurately, and these inaccuracies can be higher than for low-income households.

Investments (I_t), exports (X_t) and imports (M_t) data are taken from the National Accounts, and stationarized with the same trend z_t. Even after such detrending, exports and imports series include an upward trend. This may be due to the fact that the Czech economy is more integrated into the world economy and increasingly open. To obtain a stationary series we use a simple linear trend. This also has direct implications for GDP series. Consequently, for the aggregate GDP series (Y_t), we employ a linear trend in addition to the stochastic trend to obtain the stationary series.

Fiscal variables are solely from the National Accounts and foreign sector variables are quarterly data taken from the Eurostat database.

3.2 Steady-states

In order to use the above mentioned approximation, steady states of the model have to be defined properly. The solution of the model in steady states is crucial for determining the long-term dynamics of the model. It also checks the consistency of the key variables and parameters. Variables used in the model can be either constant or constantly growing on their equilibrium paths.[3] The main steady state equations follows.

Steady-state technological growth is a crucial factor for the determination of the interest rate at steady-state. Interest rates in domestic and foreign economy are assumed to develop in line.

$$\mu_z = \frac{\bar{R}}{1+\bar{\pi}}$$
$$1 = \frac{\bar{R}}{\bar{R}^*}$$
$$\bar{R} = \frac{1}{\beta}$$

GDP identity holds.

$$\bar{y} = \bar{c} + \frac{\bar{g}^C}{\bar{P}} + \bar{i} + \bar{x} - \bar{m}$$

Consumption and investments price indices are weighted averages of domestic and foreign prices with respective shares and elasticity parameters.

$$\bar{P}^C = \left[(1-\mu_{cm})(\bar{P})^{\frac{\theta_c}{\theta_c-1}} + \mu_{cm}(\bar{P}^m)^{\frac{\theta_c}{\theta_c-1}}\right]^{\frac{\theta_c-1}{\theta_c}}$$
$$\bar{P}^I = \left[(1-\mu_{im})(\bar{P})^{\frac{\theta_i}{\theta_i-1}} + \mu_{im}(\bar{P}^m)^{\frac{\theta_i}{\theta_i-1}}\right]^{\frac{\theta_i-1}{\theta_i}}$$
$$\bar{P}^m = \bar{S}\bar{P}^*$$

Shares of domestically produced and imported consumption goods depend on relative

[3] Those denoted with capital letters have growing steady states while the others are stationary. For differences in notations see Section 2.1.

prices and elasticity parameters.

$$\bar{c}^d = (1-\mu_{cm}) \left(\frac{\bar{P}}{\bar{P}^C}\right)^{\frac{1}{\theta_c - 1}} \bar{c}$$

$$\bar{c}^m = \mu_{cm} \left(\frac{\bar{P}^m}{\bar{P}^C}\right)^{\frac{1}{\theta_c - 1}} \bar{c}$$

$$\bar{i}^d = (1-\mu_{im}) \left(\frac{\bar{P}}{\bar{P}^I}\right)^{\frac{1}{\theta_i - 1}} \bar{i}$$

$$\bar{i}^m = \mu_{im} \left(\frac{\bar{P}^m}{\bar{P}^I}\right)^{\frac{1}{\theta_i - 1}} \bar{i}$$

$$\bar{c}^N = \frac{(1-\bar{\tau}^w)\bar{W}\bar{N}^N + \bar{\tau}^b \bar{W}(\bar{N}^N - \bar{L}^N) + \bar{TR}}{\bar{z}(1+\bar{\tau}^c)\bar{P}^C}$$

Steady-state capital stock is affected mainly by investments at steady-state and the depreciation rate. Capital demand is derived from capital stock using the utilization rate.

$$\bar{k}^s = \left(\frac{\mu_z - 1 + \delta}{\mu_z}\right) \bar{i}$$

$$\bar{k} = \bar{u}\bar{k}^s \frac{1}{\mu_z}$$

Wages are determined as a result of negotiations between firms and households. Total amount of labour supply is the sum of that offered by both Savers and Spenders.

$$\bar{N} = \alpha_R \left(\frac{1}{\pi^w}\right)^{\frac{1}{\psi_n}} \bar{N}^r + (1-\alpha_R)\bar{N}^n$$

Government revenues and expenditures are the results of average incomes and outlays of considered variables. Government revenues are derived from taxes on wages, consumption tax and corporate tax. Government expenditures consist of spending on consumption, social benefits/transfers and other expenditure.

$$\bar{gr} = \bar{\tau^w} \frac{\bar{W}}{\bar{z}} \bar{L} + \bar{\tau}^c (\bar{P^C}\bar{c} + \bar{g}^C) + \bar{\tau}^k (\bar{R^k}\bar{k} + \bar{q})$$

$$\bar{ge} = \bar{g}^C + \bar{\tau}^b \frac{\bar{W}}{\bar{z}} (\bar{N} - \bar{L}) + \bar{TR} + \bar{g}^O$$

Profit is derived from production minus total costs, i.e. labour and capital expenses.

$$\bar{q} = \bar{P}\bar{y} - \frac{\bar{W}}{\bar{z}}\bar{L} - \bar{R}^k \bar{k}$$

Steady-state of Tobins' Q is given by:

$$\bar{TQ} = \frac{\beta \bar{R}^k}{1-\beta(1-\delta)} \left(1 - \bar{\tau}^k\right)$$

Steady-state values of the variables are critical for writing non-linear equations in log-linear approximation form. Appendix A contains the whole log-linearized system.

3.3 Calibration and estimation

To assign values for the model parameters, we have used calibration technique mostly by using information available in the data. As parameters and steady-state values of the variables are linked through model equations, we are often able to come up with reasonable parameter values using the data. Also, we referred to the literature, relied on expert judgements and employed common sense.

The source of long-term growth in the model is technology that is given exogenously. It is crucial to define steady-state technology growth as it is also the steady-state (potential) growth of the real economy. After discussion with experts and data analysis, we come up with 2% of long-term annual technology growth, meaning $\mu_z = 1.005$. This steady-state growth rate is relevant for GDP and its components.

Consistent with the targeted inflation rate by the Czech National Bank, we assign $\bar{\pi}$ to be 1.005, that is 2% annual steady-state price increase. This implies that we set discount factor as $\beta = 0.99$. Such value for the discount factor can be found in the literature, for instance in Hansen (1985).

In order to find depreciation rate, we refer to physical capital accumulation equation. We use steady-state values for the physical capital and investments and derive the depreciation rate δ to be 0.01. It implies about 4% of annual depreciation. Another parameter or production process is η. We take $\eta = 0.53$ that plays an important role for the solution of the steady-state system of equations enabling a reasonable solution.

Regarding the value of habit formation parameter h_R, there are many studies that considers it to be within 0.6–0.8 range (Christiano et al., 2005, Adolfson et al., 2007a, Forni et al., 2007). Hence, we set it to be 0.8 in our model. The labour supply elasticity ψ_n is calibrated to 1.55 taken from Burriel et al. (2010).

Share of Saver (or Ricardian) households α_R is set to 32% based on households' income distribution analysis and expert opinion. Using the value of α_R, from Households Budget Survey analysis we can find the ratio of Saver household's consumption to average consumption s_R to be around 1.55, which means that a representative Saver household consumes approximately twice as much as a representative Spender household.

For price ξ_p and wage ξ_w rigidity parameters we follow Adolfson et al. (2007a) which are around 0.9 and 0.7, respectively. We take ξ_p to be the same. However, for ξ_w we need to transform parameter due to the difference in modeling framework. We assumed in our model that only a fraction Savers can adjust their wages. In order to have 70% non-adjusting households in total we need to have very low ξ_w for Savers. Hence, we take $\xi_w = 0.2$ in our model.

Steady-state values for relative prices are derived form the data to match their sample means. Doing so, we have $\frac{\bar{P}}{\bar{P}^C} = 1.01$, $\frac{\bar{P}^m}{\bar{P}^C} = 0.95$, $\frac{\bar{P}}{\bar{P}^I} = 1.03$ and $\frac{\bar{P}^m}{\bar{P}^I} = 0.98$. Also, for steady-state values of the implicit tax rates, we refer to data and match with their sample means. We find that $\bar{\tau}^w = 0.23$, $\bar{\tau}^c = 0.16$ and $\bar{\tau}^k = 0.15$. Implicit rate of unemployment benefits has its value at steady state $\bar{\tau}^b = 0.001$.

The share of foreign demand for domestic production on total exports s_{xc} was derived from the share of steady state values of foreign demand for consumption and domestic export as $s_{xc} = \frac{\bar{C}_t^{m*}}{\bar{X}} = 0.70$. Similarly, the share of imported goods for consumption on total imports s_{mc} was calculated as the share of steady states of imported consumption and domestic import as $s_{mc} = \frac{\bar{C}_t^m}{\bar{M}_t} = 0.78$.

Further, the share of imported consumption on total consumption μ_{cm} and the share of imported investment on total investment μ_{im} were also assigned from the steady state values of consumption and investment respectively. The shares were derived as $\mu_{cm} = \frac{\bar{C}_t^m}{\bar{C}_t} = 0.22$ and $\mu_{im} = \frac{\bar{I}_t^m}{\bar{I}_t} = 0.32$.

The substitution elasticity parameter between foreign and domestic consumption goods θ_c (and also θ_{c*}) was set with respect to steady-state system and its solution. As a result, we set both these values amounting to 0.3. Similarly we derived the elasticity parameter between foreign and domestic investment goods θ_i (and θ_{i*}) equal to 0.2.
We faced difficulties to assign a value to the wage mark-up parameter θ_w. The range of values of this parameter in literature is quite high. Moreover, we were not able to set its value from the data unequivocally. Therefore, we calibrate this parameter to value 2 because this value stabilizes our model as the best. In the future, it would be more appropriate to estimate it with Bayesian estimation techniques.

The adjustment cost function parameter κ is estimated as a combined parameter from the log-linearized investment equation employing restricted OLS. The restrictions were imposed on the coefficients which combined already calibrated parameters β and μ_z. Then the parameter κ with value 11 is calculated from the estimated coefficient and known parameters.

The monetary policy parameters ϕ_r, λ_π and λ_y were set according to previous version of the model, i.e. calibrated from Štork et al. (2009). Last but not least, we employe restricted OLS to estimate the parameter ω_{rkz} from the log-linearized capital yields equation. The estimated coefficient is equal to 12.72.

To replicate the behavior of the exogenous variables in the government and foreign economy sectors, we estimated their AR(1) processes. All of the calibrated and estimated parameters are summarized in Appendix B.

4 Simulations

Simulation results illustrate the main responses to shocks into various variables. To test the stability of the model we focus on macroeconomic shocks, namely the external demand shock, price shock, interest rate shock, as well as fiscal shocks, such as changes in the tax rate on consumption, wages and government spending. All shocks are temporary and unanticipated unit shocks. Illustrations in Appendix C show log deviations from the steady-state.[4]

Foreign demand shock

Foreign demand shock is defined as a faster growth of the Eurozone and is illustrated in Figure 1. Looking at the results, we can generally distinguish between primary and secondary effects. The primary effect of higher external demand naturally results in higher exports with a positive effect on domestic GDP thanks to higher production. It has further consequences on labour market, where employment and wages tend to grow. Higher export activity induces higher imports for production that to some extent limit the positive impact on GDP. However, the resulting effect on foreign trade is positive and the exchange rate appreciates.

Secondary effects are those induced by higher domestic production primarily for exports. Better conditions on the labour market and somewhat higher wages motivate consumers to supply their workforce. Higher household income supports private consumption and thus also GDP. Positive demand effect from outside and the domestic economy pushes domestic prices up. Because of that consumers tend to prefer imported goods to domestic to the extend allowed by the substitutability of consumption goods.

Higher prices then play role in wage negotiations, where consumers (employees) seek compensation for higher inflation. Interest rates affected by the decision of the Central Bank react to price developments and higher GDP growth by rising, which tends to limits investment activity. However, better economic conditions and higher foreign demand overweighs and investments increase.

Positive effects can also be observed on the side of government revenues in higher tax collection from wages, consumption and profit. Marginal decrease in government spending can be explained by limited unemployment, which means lower benefits outlays.

Price shock

In a similar way, we can also infer the effects of other shocks. The price shock in Figure 2 represents a situation in the domestic economy when the consumer price index

[4]Note that in graphs, shocked variables are depicted with black line.

increases by 1p.p. The primary effect of higher price level drags down the consumption demand of both, Spenders and Savers. The latter decrease demand more as they are more sensitive to prices. Consumption demand is also shifted towards imported goods that are cheaper than domestic production. Volumes of imports therefore increase.

As a consequence, lower domestic production acts as a drag on both labour demand and wages. Firms also limit their investment activities. Higher domestic prices affect the competitiveness of exports, which grow at a slower pace. All these factors cause a decrease in the GDP growth rate. Interest rates reflect the worsened performance of the economy, and this should diminish the impact of the temporary inflation shock would be diminished.

Negative growth also affects the government's tax income and unemployment benefits raises government expenditures. Although the latter effect is limited, the overall impact on budget balance is negative.

Interest rate shock

Higher interest rates (Figure 3) limit domestic investment activity with negative consequences on GDP growth. Firms tend to lower wages and demand for labour in order to reduce their marginal costs. Prices tend to decrease, which would on the other hand supports consumption. However, that effect is not enough to compensate for the lower income of households. Moreover a higher interest rates motivates Savers to defer their spending and consumption falls.

Foreign trade also contributes to the overall negative effect on GDP. The exchange rate tends to appreciate, which reduces the prices of imports. As a result, consumption switches to imported goods rather than domestically produced ones. But the impact on imports is rather ambiguous. Higher imports for consumption are only temporary, soon the income effect of lower wages and thus lower budget constraint dominates. This together with lower investment imports cause decrease in total import. As the interest rate returns to its pre-shock value, also imports stabilizes. Somewhat worse competitiveness of domestic production on foreign market stands behind lower exporting activity.

As a result, the government also collects less tax revenue and adjusts its expenditure in response.

Tax rate on consumption shock

The first fiscal shock analyzed is a unit shock into the tax rate of consumption (Figure 4) which directly affects consumption demand by making goods more expensive. As a result, households tend to consume less domestically produced and imported goods because all goods are subject to this higher tax. Lower demand for domestic production causes the price level measured by core inflation to decline slightly.[5] Exchange rate tends to appreciate making the situation worse for export activity.

As it was the case in previous scenarios, the labour market reacts by slightly lower wages and lower demand for workers. But these effects are rather negligible. The same also applies to the Central Bank's reaction to reduced economic activity by granting lower interest rates.

Government revenues provide a positive impact. Tax income increases due to a higher tax rate on consumption despite the fact that volume of consumption decreases.

Tax rate on wages shock

Higher taxes on wages illustrated in Figure 5 directly allow government to collect higher taxes and to increase government revenues. This tax rate has an impact on households and their income, but they are motivated to seek higher wages as a compensation for higher taxation. Part of this burden is borne by firms, respectively their marginal costs. This induces higher inflation. The effect on consumption is negative. There is only a small decrease in interest rates as a result of opposite effects - slightly higher inflation and decline in output gap growth.

Imports decline as a result of lower demand from households. Exports tend to decrease too as a result of higher costs of labour at the beginning and mild appreciation of the exchange rate. The first is induced by wage negotiations with employees that seek for higher wages to compensate for taxation.

Government consumption shock

A positive shock into government consumption (Figure 6), increases government outlays, as it is an internal part of them. The same argument is valid for the positive impact on GDP. To satisfy higher demand for production, firms tend to hire more workers and wages also have a tendency to grow. This further supports private consumption and GDP.

However, these positive effects on demand push the price level up and, consequently, also interest rates. This also means a lower level of investment into the economy.

Foreign trade witnesses quite ambivalent reaction of imports. As in the case of interest rate shock, consumption substitutes domestic goods for imported ones as import

[5] One should bear in mind that the effect shown here is on core inflation, i.e. this inflation does not include administrative changes and changes in taxation as the whole CPI does. CPI thus would be naturally affected by higher tax rate.

prices go down due to the exchange rate appreciation. As soon as this effect diminishes more persistent decline in investment growth dominates, which results in decrease in imports. Prices of exports increase with higher price level and exchange rate appreciation. Export volumes decrease. The contribution to GDP growth from net exports is thus rather negative. Summing up all the factors, the final effects on GDP are positive.

From a fiscal perspective, the increase in expenditure is obvious. Revenues are also higher due to higher consumption and wages, but the overall effect on the balance tends to be negative.

5 Conclusion

In this paper, we further extend the previous dynamic general equilibrium model of the Czech economy in order to improve its quality and interpretability for the purpose of simulations and forecast. The main value added is that the model works with more macroeconomic variables, such as full expenditure structure of GDP. The cost of such an extension is reflected in the greater complexity of the model and in additional equations, which create additional channels and effects, making the background story more complex and less transparent.

We have built on the previous DSGE model at the Ministry of Finance. We developed the existing model in two ways. First, we decompose net exports into exports and imports. This allows us to better understand determinants of foreign trade. Second, as far as domestic demand is concerned, consumption and investment are now introduced into the model separately, which was not the case with the previous version. Here, the first modification is quite straightforward built up on the previous version, the second requires extensive revisions throughout the model. It was necessary to include additional variables, such as capital stock and capital demand. This affects all parts of the model.

We also focus on data description and the model solution. We show the solution of the model in steady state, enlist a full set of log-linearized equations and describe crucial parameters.

In the analytical part of the paper, we show results of simulations using impulse response analysis. As an illustration we have chosen several macroeconomic and fiscal shocks, namely external demand shock, price shock, interest rate shock, as well as innovations to the tax rate on consumption, wages and government spending.

While working on the current extension, we have found potential questions for future research in this area. First, we conjecture that some additional labour market imperfections should be reflected (such as in Moyen and Sahuc (2004), Trigari (2004) and Stevens (2007)). Second, it will be useful to explore different complex estimation methods for parameters, such as Bayesian MLE.

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Appendix A: Log-linearized equations

The non-linear model equations are log-linearized around steady states using the Taylor approximation. Employing this method, the general equilibrium condition equation $x_t = f(x_{t-1}, z_t)$ for stationarized variables x and z can be rewritten as:

$$\hat{x}_t = f_x^{'}(\bar{x}, \bar{z})\hat{x}_{t-1} + \frac{\bar{\bar{z}}}{\bar{x}} f_z^{'}(\bar{x}, \bar{z})\hat{z}_t, \tag{59}$$

where $\bar{x}$ and $\bar{y}$ are steady state values and $\hat{x}_t$, $\hat{x}_{t-1}$ and $\hat{z}_t$ denote the log-deviations from steady state. The log-linearized model equations are summarized bellow. To solve the log-linearized model we stick to a method described in Uhlig (1997).

Consumption

Log-linearizing equation (15) around steady-state[6]:

$$(1 + h_R)\hat{c}_t^R = \mu_z(\hat{c}_{t+1}^R + \hat{\mu}_{zt+1}) + \tfrac{h_R}{\mu_z}(\hat{c}_{t-1}^R - \hat{\mu}_{zt}) - \tfrac{\mu_z - h_R}{\mu_z}[\hat{R} + \hat{P}_t^C - \hat{P}_{t+1}^C + \tfrac{\bar{\tau}^c}{1+\bar{\tau}^c}(\hat{\tau}_t^c - \hat{\tau}_{t+1}^c)] \tag{60}$$

From (16):

$$\begin{aligned} \hat{c}_t^N + \hat{P}_t^C + \tfrac{\bar{\tau}^c}{1+\bar{\tau}^c}\hat{\tau}_t^c = \tfrac{(1-\bar{\tau}^w)\bar{W}\bar{N}^N}{(1+\bar{\tau}^c)\bar{P}^C\bar{C}^N}(\hat{W}_t - \tfrac{\bar{\tau}^w}{1-\bar{\tau}^w}\hat{\tau}_t^w) + \tfrac{\bar{\tau}^b\bar{W}(\bar{N}^N - \bar{L}^N)}{(1+\bar{\tau}^c)\bar{P}^C\bar{C}^N}\hat{\tau}_t^b - \\ - \tfrac{\bar{\tau}^b\bar{W}\bar{L}^N}{(1+\bar{\tau}^c)\bar{P}^C\bar{C}^N}\hat{L}_t^N + \tfrac{\bar{TR}}{(1+\bar{\tau}^c)\bar{P}^C\bar{C}^N}\hat{tr}_t - \tfrac{\bar{\tau}^b\bar{W}(\bar{N}^N - \bar{L}^N)\bar{\tau}^b\bar{W}(\bar{N}^N - \bar{L}^N) + \bar{TR}}{(1+\bar{\tau}^c)\bar{P}^C\bar{C}^N}\hat{z}_t \end{aligned} \tag{61}$$

From (17):

$$\hat{c}_t = \alpha_R s_R \hat{c}_t^R + (1 - \alpha_R s_R)\,\hat{c}_t^N \tag{62}$$

From (19) and (20):

$$\hat{c}_t^d = \frac{1}{\theta_c - 1}\left(\hat{P}_t - \hat{P}_t^C\right) + \hat{c}_t \tag{63}$$

$$\hat{c}_t^m = \frac{1}{\theta_c - 1}\left(\hat{P}_t^m - \hat{P}_t^C\right) + \hat{c}_t \tag{64}$$

Investment and capital

From (22):

$$\hat{i}_t = \frac{1}{1 + \beta\mu_z}\left(\hat{i}_{t-1} - \hat{\mu}_{z,t}\right) + \frac{\beta\mu_z}{1 + \beta\mu_z}\left(\hat{i}_{t+1} + \hat{\mu}_{z,t+1}\right) + \frac{1}{\kappa\mu_z^2(1 + \beta\mu_z)}\left(\hat{TQ}_t - \hat{P}_t^I\right) \tag{65}$$

[6]Fernández-Villaverde and Rubio-Ramirez (2006) provide detailed notes on log-linearization.

From (7):

$$\begin{aligned}&\hat{TQ}_t - \beta\left(1-\delta\right)\hat{TQ}_{t+1} - \left[1-\beta\left(1-\delta\right)\right]\hat{R}^k_{t+1} + \\ &+\left[1-\beta\left(1-\delta\right)\right]\tfrac{\overline{\tau}^k}{1-\overline{\tau}^k}\hat{\tau}^k_{t+1} + \hat{R}_t = 0\end{aligned} \tag{66}$$

From (25) and (26):

$$\hat{i}^d_t = \frac{1}{\theta_i - 1}\left(\hat{P}_t - \hat{P}^I_t\right) + \hat{i}_t \tag{67}$$

$$\hat{i}^m_t = \frac{1}{\theta_i - 1}\left(\hat{P}^m_t - \hat{P}^I_t\right) + \hat{i}_t \tag{68}$$

From (8):

$$\hat{R}^k_t = \frac{a''(\overline{u})}{a'(\overline{u})}\hat{u}_t + \frac{\overline{\tau}^k}{1-\overline{\tau}^k}\hat{\tau}^k_t + \hat{P}_t \tag{69}$$

From (24):

$$\hat{u}_t = \hat{k}_t - \hat{k^s}_t + \hat{\mu}_{z,t} \tag{70}$$

From (23):

$$\hat{k}^s_{t+1} = \frac{1-\delta}{\mu_z}\hat{k^s}_t + \frac{\mu_z - (1-\delta)}{\mu_z}\hat{i}_t - \frac{1-\delta}{\mu_z}\hat{\mu}_{z,t} \tag{71}$$

Prices

New prices are set according to (33). To get rid of infinite sums in this equation, we refer to the methodology described in McCandless (2008). Combining (33) and (34), and after log-linearizing, we obtain an aggregate domestic price equation *alias* the Philips equation:

$$\hat{P}_t = \frac{\xi_p}{1+\beta\xi_p^2\mu_z}\hat{P}_{t-1} + \frac{\beta\xi_p\mu_z}{1+\beta\xi_p^2\mu_z}\hat{P}_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p\mu_z)}{1+\beta\xi_p^2\mu_z}\hat{MC}_t \tag{72}$$

From (21):

$$\hat{P}^C_t = (1-\mu_{cm})\left(\frac{\overline{P}}{\overline{P}^C}\right)^{\frac{\theta_c}{\theta_c-1}}\hat{P}_t + \mu_{cm}\left(\frac{\overline{P}^m}{\overline{P}^C}\right)^{\frac{\theta_c}{\theta_c-1}}\hat{P}^m_t \tag{73}$$

Consumer price inflation rate is given by:

$$\hat{\pi}^c_t = \hat{P}^C_t - \hat{P}^C_{t-1} \tag{74}$$

From (27):

$$\hat{P}^I_t = (1-\mu_{im})\left(\frac{\overline{P}}{\overline{P}^I}\right)^{\frac{\theta_i}{\theta_i-1}}\hat{P}_t + \mu_{im}\left(\frac{\overline{P}^m}{\overline{P}^I}\right)^{\frac{\theta_i}{\theta_i-1}}\hat{P}^m_t \tag{75}$$

From (42):

$$\hat{P}^m_t = \hat{S}_t + \hat{P}^*_t \tag{76}$$

Wages

To obtain a log-linearized wage evolution equation, we perform few manipulations. First, we plug a wage-setting equation (38) into aggregate wage equation (37). Second, we apply log-linearization, and then, eliminating sums in this equation, we refer to the methodology described in McCandless (2008). Last, to get rid of j index from the equation obtained, we use a log-linearized version (39) for N_{jt}. With straightforward manipulation the following aggregate wage evolution equation obtains:

$$\left\{1+\left[1-\alpha_R\left(1-\xi_w\right)\right]\beta\xi_w\left(\frac{1}{\bar{\pi}_w}\right)^{1-\theta_w}+\left(1-\beta\xi_w\right)\left[\psi^N\theta^w-\alpha_R\left(1-\xi_w\right)\left(\frac{\theta^w}{\theta_w-1}\right)^{1-\theta_w}\left(\psi^N\theta^w+1\right)\right]\right\}$$
$$\hat{W}_t=\beta\xi_w\hat{W}_{t+1}+\left[1-\alpha_R\left(1-\xi_w\right)\right]\left(\frac{1}{\bar{\pi}_w}\right)^{1-\theta_w}+\left[1+\theta_W\psi_N\left(1-\beta\xi_w\right)\right]\hat{W}_{t-1}+\alpha_R\left(1-\xi_w\right)\left(\frac{\theta^w}{\theta_w-1}\right)^{1-\theta_w}$$
$$\left(1-\beta\xi_w\right)\psi_N E\left(\hat{L}_t\right)-\left(1-\xi_w\right)\left(\frac{\theta^w}{\theta_w-1}\right)^{1-\theta_w}\left(1-\beta\xi_w\right)\frac{\bar{N}}{\bar{N}^R}\hat{N}_t \quad (77)$$

Foreign sector

From (40) assuming $s_{mc}\equiv\frac{\overline{C}^m}{\overline{M}}$ to be the share of consumption goods imports out of total imports at steady state:

$$\hat{m}_t=s_{mc}\hat{c}_t^m+\left(1-s_{mc}\right)\hat{i}_t^m \quad (78)$$

From (41) assuming $s_{XC}\equiv\frac{\overline{C}^{m*}}{\overline{X}}$ to be the share of consumption goods exportsout of total exports at steady state:

$$\hat{x}_t=s_{XC}\hat{c}_t^{m*}+\left(1-s_{XC}\right)\hat{i}_t^{m*} \quad (79)$$

where

$$\hat{c}_t^{m*}=\frac{1}{\theta_{c*}-1}(\hat{P}_t-\hat{S}_t-\hat{P}_t^*)+\hat{c}_t^* \quad (80)$$

$$\hat{i}_t^{m*}=\frac{1}{\theta_{i*}-1}(\hat{P}_t-\hat{S}_t-\hat{P}_t^*)+\hat{i}_t^* \quad (81)$$

Exchange rate equation stemming from uncovered interest rate parity 43takes form

$$\hat{S}_t=\omega_{ss}\hat{S}_{t-1}+(1-\omega_{ss})\hat{S}_{t+1}-\omega_{si}(\hat{R}_t-\hat{R}_t^*)-\omega_{snx}(\hat{x}_t-\hat{m}_t) \quad (82)$$

Fiscal authority

Fiscal policy equations are easy to log-linearize and their initial equations, as mentioned in Section 2.6.1 have very similar forms. So we can derive from (45):

$$\begin{aligned}\hat{gr}_t=\ & \frac{\bar{\tau}^w\bar{W}\bar{L}}{\bar{GR}}(\hat{W}_t-\hat{z}_t+\hat{L}_t+\hat{\tau}_t^w)+\frac{\bar{\tau}^c\bar{P}^C\bar{C}}{\bar{GR}}(\hat{P}_t^C+\hat{c}_t)+\frac{\bar{\tau}^c(\bar{P}^C\bar{C}+\bar{G}^C)}{\bar{GR}}\hat{\tau}_t^c+\frac{\bar{\tau}^c\bar{G}^C}{\bar{GR}}\hat{g}_t^c+\\ & +\frac{\bar{\tau}^k\bar{R}^k\bar{K}}{\bar{GR}}(\hat{R}_t^k+\hat{k}_t)+\frac{\bar{\tau}^k(\bar{R}^k\bar{K}+\bar{Q})}{\bar{GR}}\hat{\tau}_t^k+\frac{\bar{\tau}^k\bar{Q}}{\bar{GR}}\hat{q}_t\end{aligned} \quad (83)$$

Government expenditures from (48)

$$\begin{aligned}\hat{ge}_t = \quad & \frac{\bar{G}^c}{\bar{GE}}\hat{g}^c_t + \frac{\bar{\tau}^b\bar{W}(\bar{L}^N-\bar{N}^N)}{\bar{GE}}\hat{\tau}^b_t + \frac{\bar{\tau}^b\bar{W}\bar{L}^N}{\bar{GE}}\hat{L}^N_t - \frac{\bar{\tau}^b\bar{W}(\bar{L}^N-\bar{N}^N)+\bar{TR}}{\bar{GE}}\hat{z}_t + \\ & + \frac{\bar{TR}}{\bar{GE}}\hat{tr}_t + \frac{\bar{G}^o}{\bar{GE}}\hat{g}^o_t\end{aligned} \tag{84}$$

And finally government debt (49)

$$\hat{b}_{t+1} = \frac{\bar{GE}}{\bar{B}}(\hat{ge}_t - \hat{gr}_t) + \hat{R}_t + \hat{b}_t - (\hat{z}_t - \hat{z}_{t-1}) \tag{85}$$

Monetary authority

The monetary policy rule specified in equation 50 takes form

$$\hat{R}_t = (1-\phi_r)[\lambda_\pi\hat{\pi}_t + \lambda_y\hat{y}_t] + \phi_r\hat{R}_{t-1} \tag{86}$$

Appendix B: Parameters

Parameter	Description	Value
α_r	share of Ricardians' households	0.32
β	discount factor calculated according to technological growth (2% per year assumption)	0.99
δ	quarterly depreciation rate of capital	0.01
η	production function parameter	0.53
θ_c	parameter of substitution between domestic and foreign goods	0.30
θ_{c*}	parameter of substitution between domestic and foreign goods in foreign economy	0.30
θ_i	parameter of substitution between domestic and foreign investments	0.20
θ_{i*}	parameter of substitution between domestic and foreign investments in foreign economy	0.20
θ_w	parameter of substitution between differentiated labour	2
κ	parameter of investment adjustment costs function	11
μ_{cm}	share of imported consumption on total consumption	0.22
μ_{im}	share of imported investments on total investments	0.32
μ_z	technological growth calculated as labour productivity growth	1.005
ξ_p	Calvo pricing parameter, a fraction of firms that do not re-set their prices in respective period	0.90
ξ_w	Wage-Calvo parameter, a fraction of households that are able to re-negotiate their wages	0.20
ρ_{μ_z}	technological growth AR(1) parameter	0.81
ψ_n	inverse of substitution of labour supply	1.55
ϕ_r	smoothing parameter in MP rule	0.52
λ_π	inflation weight in MP rule	1.50
λ_y	output gap weight in MP rule	0.50
h_r	habit formation of Ricardians' households	0.8
s_{mc}	share of imported goods for consumption on total imports	0.78

Parameter	Description	Value
s_r	share of Ricardian households consumption on average consumption	1.55
s_{xc}	share of consumption goods in total exports	0.70
ω_{ggc}	government consumption AR(1) parameter	0.88
ω_{go}	other government expenditures AR(1) parameter	0.78
ω_{ss}	weight of lagged value in exchange rate equation	0.50
ω_{ss}	weight of interest rate differential in exchange rate equation	0.50
ω_{ss}	weight of current balance effect in exchange rate equation	0.25
ω_{rkz}	capital utilization parameter	12.72
ω_{tc}	consumption tax AR(1) parameter	0.60
ω_{tw}	income tax AR(1) parameter	0.61
ω_{tk}	capital tax AR(1) parameter	0.91
ω_{tb}	rate of benefits AR(1) parameter	0.91
ω_{tr}	transfers AR(1) parameter	0.92

Appendix C: Simulation results

Figure 1: Foreign demand shock

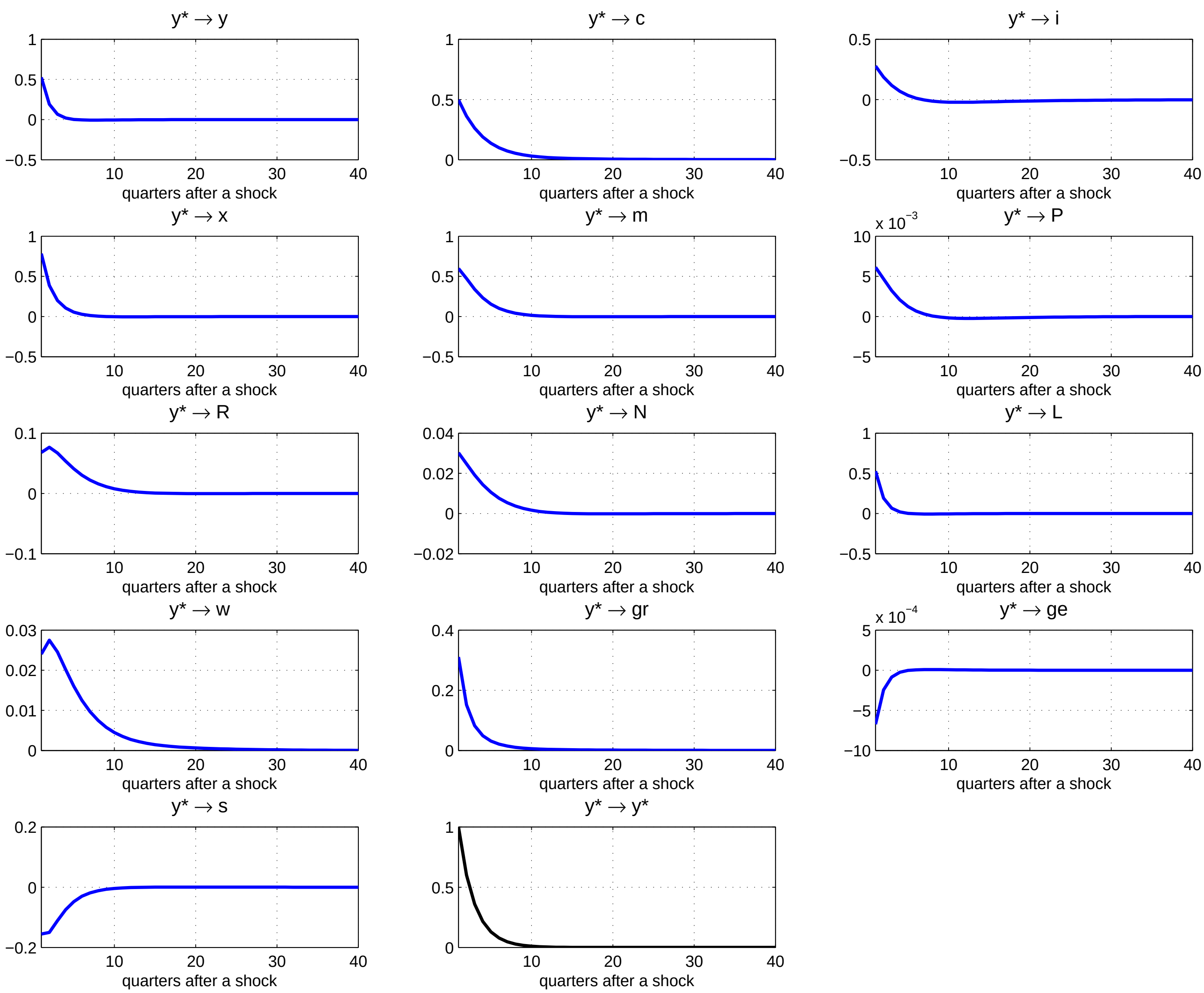

Figure 2: Price shock

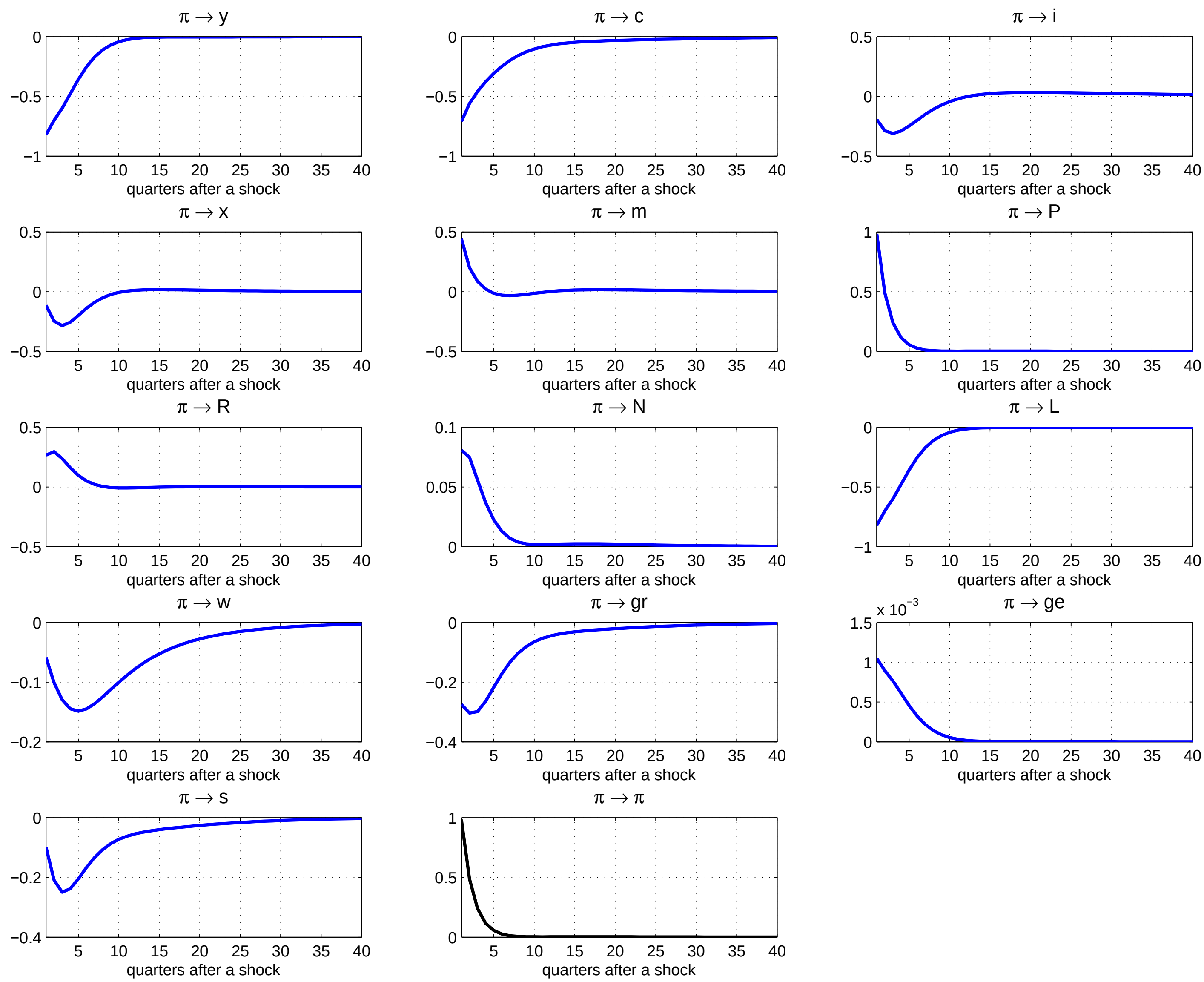

Figure 3: Interest rate shock

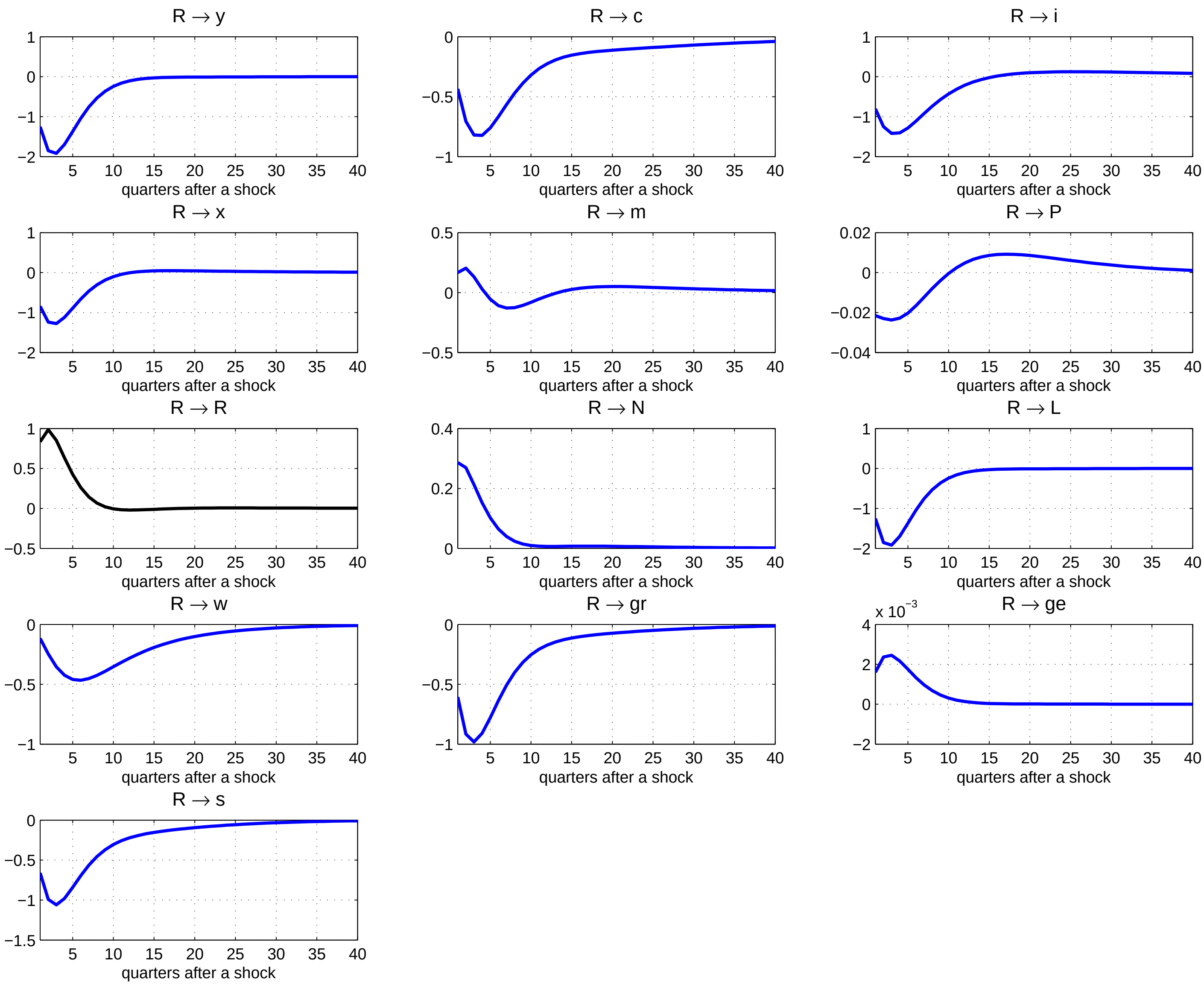

Figure 4: Tax rate on consumption shock

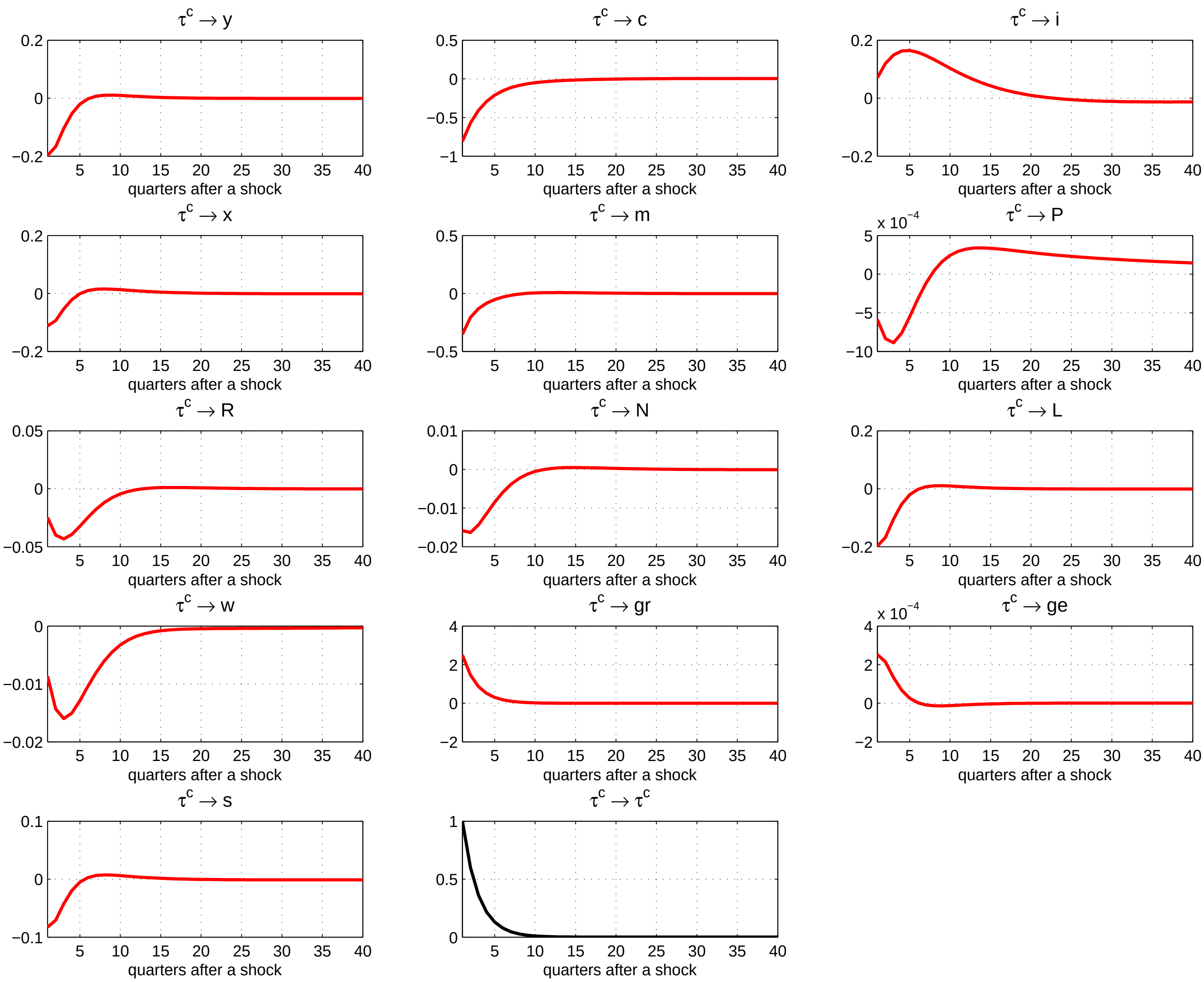

Figure 5: Tax rate on wages shock

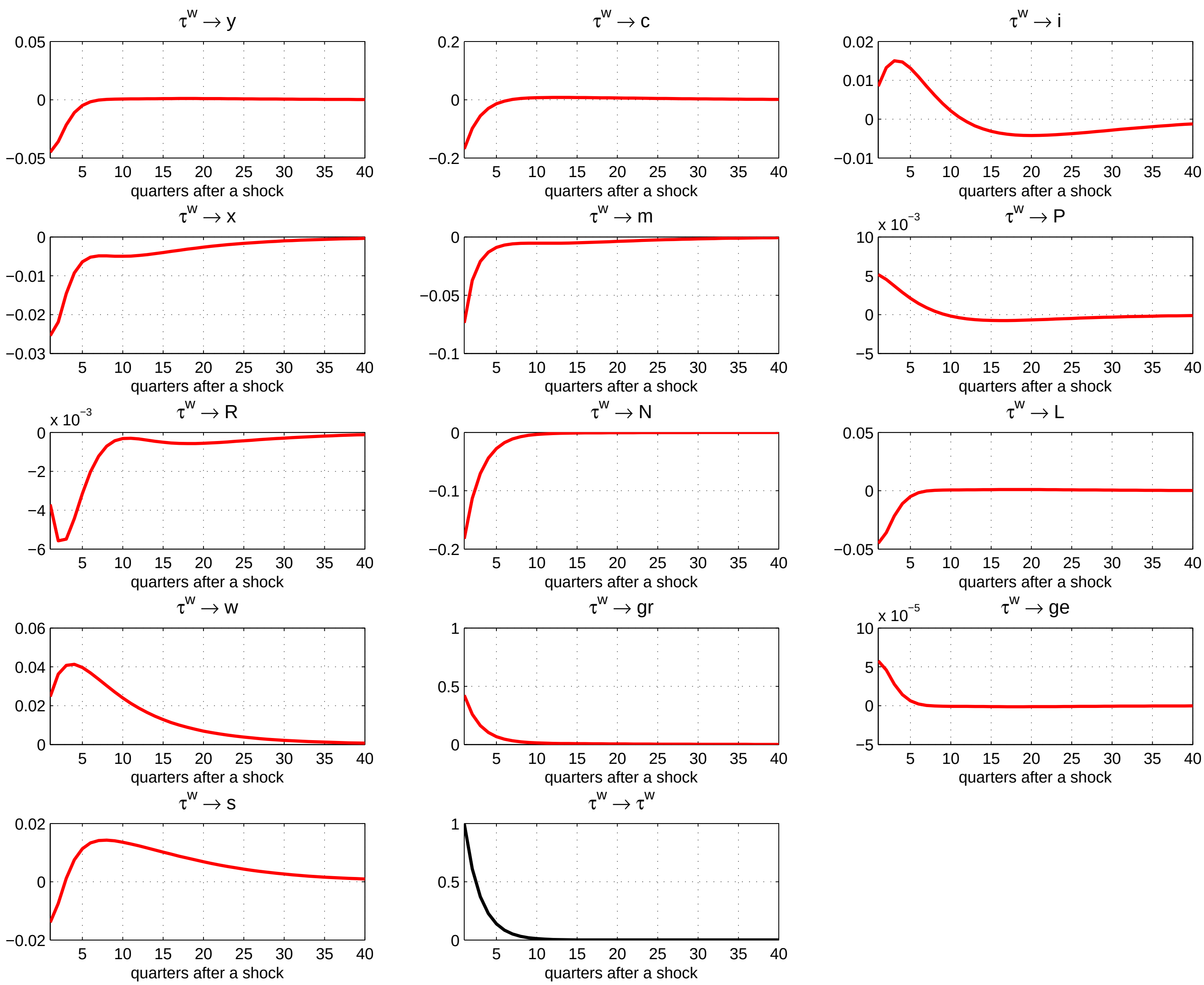

Figure 6: Government expenditure shock