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Cyclical adjustment of the government balance

Lukáš Lang, Jan Mareš

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Information papers of the Ministry of Finance of the Czech Republic are designed to provide information on current issues related to economic policy, with particular emphasis on fiscal policy. Information papers are referred by a Ministry of Finance internal opponent. The decision to publish the paper by the relevant department is based on the judgement of the head of the particular unit responsible for dealing with the given issue.

List of Abbreviations

AWG .... Working Group on Ageing Populations and Sustainability
ADF .... Augmented Dickey-Fuller
EC .... European Commission
ECB .... European Central Bank
EPC .... Economic Policy Committee
EU .... European Union (28 Member States coverage)
GDP .... gross domestic product
GVA .... gross domestic product
HP filter .... Hodrick-Prescott filter
MF CR .... Ministry of Finance of the Czech Republic
NAIRU .... non-accelerating-inflation rate of unemployment
NAWRU .... non-accelerating-wage rate of unemployment
OECD .... Organisation for Economic Co-operation and Development
SVAR .... Structural vector autoregression

1 Introduction

The information paper Cyclical Adjustment of the Government Balance is a contribution to the debate related to the legislative proposal about fiscal responsibility, approved by the government on 23 February 2015 (Resolution of the Government No. 114). Under the new fiscal responsibility rules, the Ministry of Finance of the Czech Republic (MF CR) should propose a methodology on the cyclical adjustment of the government balance in cooperation with the National Budgetary Committee. A crucial part of the process is a correction of the government balance with respect to the business cycle. This information paper introduces the most commonly used methods for the adjustment of expenditures and revenues to common economic fluctuations.

The methodology usually consists of two steps. First, it is necessary to identify the position of the economy within the economic cycle. In practice this consists of the estimation of the potential output of the economy, i.e. the output that is consistent with a steady rate of inflation, given the optimal use of the production factors. A few main methodological approaches exist. The simplest techniques of potential output estimation are based on statistical and econometric methods. They estimate potential output using various univariate filters or simple regressions. However, these approaches suffer from some technical difficulties as well as from lacking a foundation in economic theory. To some extent, this is remedied by using multivariate versions of these filters, which aim to incorporate economic relationships into the estimation process. The most complex methods then use structural economic models usually building upon the production function. The latter is nowadays widely used by policy makers and international organisations.

The Ministry of Finance conducted a background survey to supplement this study, addressing all 27 ministries of finance or economy within the EU asking about their methodology for estimating the potential output and cyclical adjustment of the government balance. From the sample of the replies the most common methodology is the production function methodology, applied in Belgium, Bulgaria, Denmark, Estonia, Finland, France, Croatia, Italy, Cyprus, Malta, Germany, Romania, Slovakia and United Kingdom, where the statistical methods also serve for complementary analysis. The main reason for using the production function methodology is the choice by the European Commission (EC) to employ a commonly agreed production function method to monitor economic developments in the Member States. Apart from the MF CR, the Czech National Bank also uses the production function method for the estimation of potential output.

The second part of the cyclical adjustment of the government balance uses the potential output and implied output gap (the difference between actual and potential output of the economy) estimated in the first part. Utilizing estimated elasticities of revenue and expenditure to the business cycle, it is then possible to adjust the government balance for the cyclical fluctuations. When it comes to the cyclical adjustment, there are two methods we explore in this study. The first is the methodology used by the Organisation for Economic Co-operation and Development (OECD), which is also used by the EC and some Member States. It is based on the elasticities for individual components periodically published by the OECD. The second approach has been introduced by the European Central Bank (ECB). It is an alternative to the OECD method focusing on elasticities to the individual macroeconomic bases (e.g. for consumption tax revenues, the cyclical development of household consumption functions as the macroeconomic base) instead of the output gap. There is more variety in the selection of the adjustment approach among the Member States than was the case with potential output. Overall, the majority of countries apply the EC approach, i.e. the methodology of the OECD. It is used almost identically by Belgium, Bulgaria, Estonia, Finland, France, Croatia, Italy, Luxembourg, Malta, Germany, Slovakia and Spain. Denmark, United Kingdom and Sweden carry out more detailed analyses, estimating their own elasticities instead of taking over the ones computed by the OECD.

The study is divided into three chapters. The following Chapter 2 discusses the estimation of potential output, with a focus on the pros and cons of the individual methods. It also applies selected methodologies to the Czech data. Chapter 3 then depicts two different ways of adjusting the cyclical government balance using the results from the preceding chapter. We conclude the results and findings in Chapter 4.

2 Potential output, output gap

Potential output and output gap are key macroeconomic variables for fiscal and monetary policy makers. Potential output is defined as the level of output consistent with the steady rate of inflation where the factors of production, including the labour force, are also all optimally employed. The output gap can be computed as the difference between actual and potential output. It is often considered a variable capturing the development of the business cycle as it reflects the balance between aggregate supply and demand. A sustained negative output gap may be a consequence of low aggregate demand and signal the need of a fiscal impulse to boost the economy. The same signal might induce a looser monetary policy. On the contrary, a positive output gap should result in a more restrictive fiscal and monetary policy. The output gap is also the key variable for the cyclical adjustment of the government balance.

Determining the output gap and potential output is not straightforward, as potential output is not directly observable, contrary to actual output, not even retrospectively. Therefore it is necessary to rely on the estimations of potential output and the output gap, which can be obtained using numerous methodologies. The basic approaches rely on using simple statistical filters or econometric estimation assuming a linear trend for potential output. More advanced methods expand the filters to include established economic relationships, such as Phillips curve or Okun's law[1]. The most complex structural methods use specifically defined economic relationships to estimate the output gap. The ultimate goal of all the methods is to decompose the time series of real output into a trend and cyclical part:

$$Y_t = Y_t^p + Y_t^c \qquad (1)$$

Where Y_t is seasonally adjusted real gross domestic product (GDP), Y_t^p potential output and Y_t^c the cycle, i.e. the output gap at time t.

Although the majority of the methods applied to the data of developed countries give a similar picture of the business cycle, the amplitude of potential output and the output gap usually differ with the methodology utilised for the estimation. The following subchapters present an overview of the approaches towards the estimation of potential output with particular emphasis on their advantages and disadvantages.

2.1 The linear trend and univariate filters

2.1.1 Linear trend

Linear trend is the simplest method for the estimation of potential output and output gap. It assumes potential output is a linear function of time t. In practice, a logarithmically transformed[2] series of real GDP enters into linear regression as a dependent variable with the time trend t and constant on the right hand side of the equation:

$$y_t = \alpha + \beta t + \varepsilon_t \qquad (2)$$

Following the estimation of the regression coefficients, we can compute the logarithm of potential output, $\boldsymbol{y_t^p}$, as $\boldsymbol{y_t^p = \alpha + \beta t}$. The residual $\boldsymbol{\varepsilon_t}$ represents the estimated output gap.

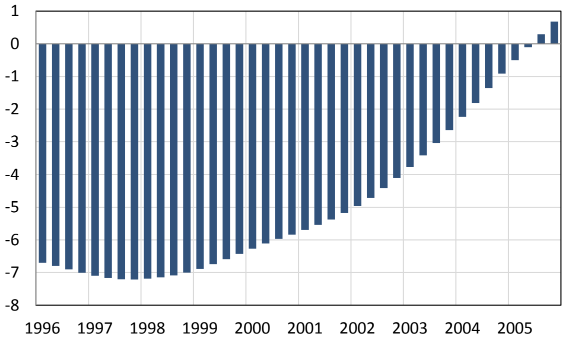
The main advantage of the linear trend method lies in the ease of its application, but it also includes severe shortcomings. Above all, the results are largely influenced by the span of the time series used for the estimation. Potential output at a specific time can differ substantially if we consider the period from the first quarter of 1996 until the fourth quarter of 2014 from the potential output estimated using linear regression and a time period from the beginning of 2005 until the end of 2014. The most significant differentials arise when the economy is in one of the extreme phases of the business cycle, a trough or peak, at the beginning or at the end of the estimation sample. Figure 2.1 illustrates this aspect showing the output gap for the fourth quarter of 2014, estimated using the series with different starting dates. Figure 2.2 then portrays the output gap in the period between the first quarter of 2005 and the fourth quarter of 2014.

[1] Phillips curve and Okun's law describe the relationship between the output gap, inflation and unemployment, respectively.

[2] Natural logarithm helps to stabilise the time series of real output and allows for easier interpretaion.

Figure 2.1: Output gap in the fourth quarter of 2014 with different starting dates for the series

(in % of potential output)

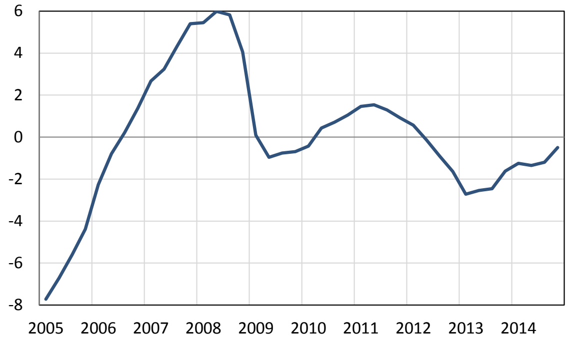


Note: The x axis shows the starting dates of the series used for estimation.

Source: MF CR.

Figure 2.2: Output gap 2005–2014 estimated using linear time trend method

(in % of potential output)



Source: MF CR.

If the period at the beginning of the sample is the trough of the business cycle, the resulting growth of potential output will be overestimated. At the same time, the output gap will be underestimated. The linear trend method also assumes a potential output growth that is constant in time. To illustrate the difficulties brought by this assumption, let us turn to the production function method described in section 2.3.1. It depicts how the growth of output can be decomposed into total factor productivity growth and the growth of the production factors – labour and capital. We can get an even more detailed perspective when we analyse the labour production factor, breaking it down further into the contribution of population growth, the change in participation rates and the average hours worked by individuals. In essence, the assumption about constant growth of potential output translates into an assumption that the growth of all these underlying series is constant as well. In reality, this assumption is hardly reasonable (de Brouwer, 1998). Changes in the underlying series may be the outcome of the labour market reforms, population ageing, or the availability of preschool childcare facilities. If the potential output follows the stochastic process, in other words it exhibits random changes of trend, the linear trend method includes the stochastic component into the estimated output gap. This implies that the resulting output gap is non-stationary, because it contains this stochastic component and its estimate may be biased (Cotis *et al.*, 2005).

The time series of real GDP growth of the Czech Republic exhibits features of the stochastic process. Table 2.1 reports the statistics from the Augmented Dickey-Fuller (ADF) applied to quarterly real GDP series 1996-2014 where the test accounted for the potential linear time trend and one lag of differences in real output in the case of real output in levels. With the differenced series of real output, the test was carried out without linear time trend and no lagged values were considered in the ADF test. Lag selection was based on the Schwarz information criterion. The first two columns of the table show that the time series of potential output is non-stationary even after considering the linear time trend in the data. On the contrary, in the differenced time series of output, we can reject the null hypothesis of non-stationarity at least at a 5% level of significance. The remaining columns report the result of Phillips-Peron unit root test which confirms the conclusions of the ADF test. Given these findings, it is advisable to assume a variable character of the potential output growth and rely on methodologies which reflect these characteristics accordingly.

Table 2.1: The results of Augmented Dickey-Fuller test on the time series of real GDP and output gap estimated using linear trend method (1996–2014)

series	ADF stat.	p-value	PP stat.	p-value
Y_t	-2.1	0.5	-1.5	0.8
ΔY_t	-3.5	0.011**	-3.5	0.009***

*Note: ** and *** denote significance on 5% and 1% level, respectively.*

2.1.2 Univariate Hodrick-Prescott filter

One of the widely utilised methods for the decomposition of a time series into its trend and cyclical components is the application of the Hodrick-Prescott (HP) filter. The trend component is extracted from the time series by minimizing the function:

$$\min_{\tau_t} \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \tag{3}$$

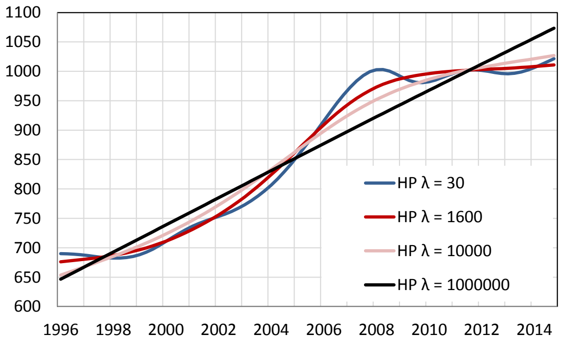
Where y_t is an observation of the time series at time t, τ_t is the trend component (potential output in our case) at time t, and λ is the so called Lagrange multiplier.

Multiplier λ affects the smoothness of the resulting trend time series. Its value is dependent on the assumptions about the relative variances of trend and the cyclical component (Hodrick & Prescott, 1997). Low values of the multiplier lead to an estimated trend which is very close to the values of the original time series. On the contrary, high values of the multiplier imply reduced sensitivity of the trend to short-term fluctuations of the observed time-series. For $\lambda \rightarrow \infty$ the filter converges towards the linear trend. Figure 2.3 illustrates this showing the trend components estimated from the series of real GDP with different values of λ. A very instructive formal solution of equation (3) is available in Reeves *et al*. (2000).

The HP filter limits the non-stationarity of the output gap discussed in the previous chapter by allowing the trend component to vary in time. A significant advantage of the HP filter also lies in its relatively easy implementation. Nevertheless, the HP filter also suffers from several drawbacks. Most importantly, it is necessary to choose parameter λ, which then affects the estimated potential output and the size of the output gap. Even though the choice of λ can significantly change the output of the filter, there is no theoretical background which would justify selection of any particular value. A widely used value of λ for quarterly data is 1600. Yearly time series usually work with the value between 6.25 and 100. It is important to note that the setting of the parameter influences the length of the estimated cycle. The size of λ around 100 for yearly data implies a cycle length of 15 years. If low values of λ around 10 are applied, the corresponding cycle length is at a maximum of 8 years (Cotis *et al.*, 2005). Although these values for different data frequencies are commonly applied, they might not reflect the variance ratio of trend and cycle components accurately. The smoothing parameter for quarterly data is questioned for example by Guay & St-Amant (1996). Running the Monte Carlo simulation, the authors show that the value $\lambda = 1600$ is optimal only when we hold very specific assumptions about the length of the business cycle and both demand and supply shocks. Harvey & Jaeger (1993) also document how the precision of the decomposition of real output on trend and the cyclical component is dependent on the input data and its characteristics.

Figure 2.3: Trend component of the GDP using HP filter with different values of the parameter λ

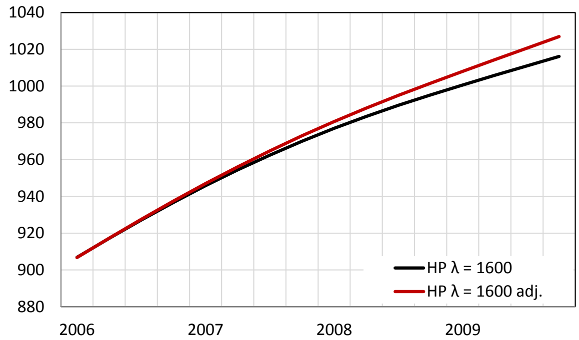
(bn. CZK, quarterly data)



Source: MF CR calculations.

Figure 2.4: Trend component of the GDP with adjusted weights of λ at the end of the sample

(bn. CZK, quarterly data)



Source: MF CR calculations.

Apart from the selection of parameter λ, the HP filter also has a serious downside because of its bias at the end of the sample. This is also known as the end-bias problem. The function minimised to decompose the trend and cycle of the time series contains the future (past) values, which are unknown at the end (beginning) of any time series. Therefore, the filter is symmetric only at times where these values are available. This shortcoming is particularly damaging at the end of the sample, which is crucial from the policy maker's perspective. The last observations are assigned a higher weight relative to the other observations (St-Amant & van Norden, 1997). The estimated potential output and output gap at the end of the sample may therefore be subject to significant revisions when the new data becomes available, especially when the economy is at the peak or at the bottom of the business cycle. To mitigate the end-point bias, forecasts of the time series may be used and the HP filter applied to an extended series. The positive impact of such a tweak, in other words, the lower bias at the end of the sample, is dependent on the reliability of the forecast. This condition limits the usefulness of this adjustment as the vast majority of forecasting methods depend upon past values and future observations may differ substantially with the occurrence of unexpected shocks.

Bruchez (2013) presents an alternative method of HP filter adjustment to end-point bias. Lower weight is given to the observations at the beginning and at the end of the sample through the different weights of the basic parameter lambda. The bias of resulting potential output series decreases and the magnitude of the cyclical component increases (Figure 2.4). The HP filter adjusted in this fashion is employed for the cyclical adjustment of the government balance by the Federal Department of Finance of Switzerland because despite the listed shortcomings, this method is transparent, relatively easy to implement and decomposes the time series symmetrically (i.e. the sum of estimated cyclical components is equal to zero).

2.1.3 Baxter-King filter

Baxter and King (1995) proposed a statistical filter which relies on the frequency analysis of data. This approach filters the very stable (low frequency) and very volatile (high frequency) components from the time series. The remainder of the time series is considered to represent the business cycle. The original version of the Baxter-King (BK) filter maintains the components fluctuating with a frequency between 6 and 32 quarters and takes the form of a 24 period weighted moving average. These filtering parameters may be adjusted to suit the analysed time series and such variability is the main advantage of the filter. On the other hand, the main disadvantage of the filter from the policy perspective is the unavailability of the trend and cycle estimates for the boundary observations. This is implied by the construction of the filter as the moving average. This may be overcome by considering the forecasted values of the time series during the filtering, but the forecast has to be quite long and as was the case with the HP filter, the estimated trend is critically dependent on the reliability of the forecasted series (Guay & St-Amant, 1996).

2.1.4 The Kalman filter

The methodology suggested by Watson (1986), also known as the method of unobservable components, is another purely statistical method for the decomposition of a time series into a trend and cyclical part. After specifying the processes both components follow, it is possible to estimate the coefficients in the system using the Kalman filter.

Watson proposed a specification based on an equation (1). In particular, a time series can be decomposed into a trend, y_t^p, and cycle, y_t^c. In case of GDP, the trend part is interpreted as the potential output and the cyclical part as the output gap. The basic equation is supplemented by an additional assumption about the individual components. Potential output is usually specified as a random walk with a drift:

$$y_t^p = \mu_t + y_{t-1}^p + \eta_t \tag{4}$$

Where μ_t is the average growth rate of potential output and η_t is a residual such that $\eta_t \sim N\left(0, \sigma_{y^p}^2\right)$. In other words, the residual is on average equal to zero and its variance is $\sigma_{y^p}^2$. Enriching Watson's original model, contemporary models assume a variable rate of potential growth which follows a random walk:

$$\mu_t = \mu_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\mu^2) \tag{5}$$

The most common specification for the cyclical part is the autoregressive process of the second order:

$$y_t^c = \phi_1 y_{t-1}^c + \phi_2 y_{t-2}^c + \nu_t \tag{6}$$

The cyclical component at time t is therefore dependent on two of its lagged values. ϕ_i are the estimated parameters and v_t is a residual, $\nu_t \sim N(0, \sigma_v^2)$. This model is then transformed into a state-space form and estimated with the Kalman filter by maximizing the likelihood function.

The method of unobservable components accommodates the characteristics of the trend and cyclical parts. The model considers the variability of potential output in time and equation (6) implies that the cyclical part, the output gap, equals zero on average. On top of that, the specifications of trend and cycle may be adjusted to reflect the characteristics of the analysed economy. Few issues, nevertheless, arise during the estimation process. It is necessary to make assumptions about the initial values of the model parameters. With a very long time series, these assumptions should not matter and the parameters eventually converge to their true values. However, if the time series are too short, the optimisation process may become difficult and the initial parameter values may affect the final estimates of the individual components.

2.2 Multivariate filters

2.2.1 Multivariate Hodrick-Prescott filter

The semi structural filter proposed in the study by Laxton & Tetlow (1992) estimates potential output with an extended version of the original Hodrick-Prescott filter. Similarly to the HP filter, its multivariate version minimises the weighted average of the output gap and the change in the rate of growth of potential output. In addition, it considers the residuals from other structural equations which contain the output gap. Such structural equations are often the Phillips curve, capturing the relationship between inflation and the output gap, and Okun's law, capturing the dependency of unemployment on output or the output gap. Supplementary information contained in the residuals increases the precision of the estimates and including structural relationships, also reflects the criticism of purely statistical methods for the estimation of potential output. The specification of the multivariate HP filter is as follows:

$$\min_{\tau_t} \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 + \sum_{t=1}^{T} \beta_t \varepsilon_{\pi,t}^2 + \sum_{t=1}^{T} \mu_t \varepsilon_{u,t}^2 \quad (7)$$

Where the basic structure and series remain the same as in section 2.1.2, but the equation is augmented by the residuals from the previously mentioned Phillips curve, $\varepsilon_{\pi,t}$, and by residuals from the relationship between output and unemployment – Okun's law, $\varepsilon_{u,t}$. Specification of the Phillips curve in the elementary setting takes the following form:

$$\pi_t = \alpha \pi_t^e + \phi(y_t - \tau_t) + \varepsilon_{\pi,t} \quad (8)$$

π_t represents the inflation at time t, π_t^e the inflation expectations and residual $\varepsilon_{\pi,t}$ then enters info the multivariate filter. The inflation expectations are often modelled as a weighted average of lagged inflation values and the influence of the output gap on inflation is delayed by one period. Phillips curve in Laxton & Tetlow (1992) is:

$$\pi_t = \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + \alpha_3 \pi_{t-3} + \alpha_4 \pi_{t-4} + \phi(y_{t-1} - \tau_{t-1}) + \varepsilon_{\pi,t} \quad (9)$$

This form of the Phillips curve is also used for the purpose of this study, but some models indeed use a different specification. The equation might include the inflation expectations from the surveys, prices of imports or values of exchange rates, as is the case for example in the model by Benes & N'Diaye (2004).

Okun's law may be expressed algebraically as:

$$u_t^c = \gamma u_{t-1}^c + \delta(y_{t-1} - \tau_{t-1}) + \varepsilon_{u,t} \quad (10)$$

In equation (10) u_t^c represents the cyclical part in the rate of unemployment at time t, while the number of lagged observations of the cyclical unemployment and output gap may be adjusted by the researcher to accommodate the characteristics of the economy.

An interesting extension of this multivariate filter may be the consideration of the capacity utilisation indicator. Examples of such adjustment to the filter may be found in Conway & Hunt (1997) or de Brouwer (1998). The data on the capacity utilisation is also collected periodically by the Czech Statistical Office. Capacity utilisation correlates closely with production output, especially when it comes to the periods of economic recessions. Even though using such data is problematic as it assumes a homogenous view of the capacity utilisation among firms and it is often focused on industrial production, it might serve as a good improving indicator of the business cycle movements. The filter is then enlarged by the following equation specifying the relationship between capacity utilisation and the output gap:

$$CU_t = \tau_{cu,t} + \psi(y_t - \tau_t) + \varepsilon_{cu,t} \quad (11)$$

Where $\tau_{cu,t}$ is the equilibrium level of capacity utilisation and $\varepsilon_{cu,t}$ residuals from the estimation, which enter the equation (7) the same way that the residuals from other equations do as $\sum_{t=1}^{T} \rho_t \varepsilon_{cu,t}^2$, where ρ_t is the weight allocated to the capacity utilisation during the minimisation process. Specification involving capacity utilisation is also used in this study. In particular, potential output is estimated using the Phillips curve from the equation (9), Okun's law as specified in equation (10) and the relationship between capacity utilisation and the output gap defined in equation (11).

Although the multivariate HP filter effectively refines the purely statistical univariate approach by considering the structural relationships while maintaining the desirable characteristics of the potential output and the output gap, its utilisation is burdened with issues similar to the original HP filter. Most importantly, it is the arbitrary choice of the weights for different components entering into the optimisation process λ, β, μ, ρ. Their construction lacks a theoretical basis and their values may also be variable in time (Cotis *et al.*, 2005).

2.2.2 Multivariate Kalman filter

The multivariate filter is an extended version of the unobserved components method. Similarly to the multivariate HP filter, the basic system of equations is enlarged by structural relationships and additional observables. Kuttner (1994) suggests extending the basic model for output gap estimation by Phillips curve. Apel & Janson (1997) then study the output gap estimation using both Phillips curve and Okun's law. This approach allows for the estimation of another very important macroeconomic variable, the non-accelerating inflation rate of unemployment (NAIRU), alongside with the potential output and the output gap. Specification of the multivariate Kalman filter is not limited to these structural relationships or their concrete specifications. Under the multivariate Kalman filter, it is possible to accommodate any model specification. This flexibility makes it easier to adapt the filter to the characteristics of the investigated economy. At the same time, the applicability of the filter depends on the correct specification of additionally considered relationships. Horn *et al.* (2006) also note the availability of standard errors of the unobserved components and forecasts among the advantages of the multivariate Kalman filter. These are, at the least, very difficult to obtain with other methodologies.

As in the case of the univariate Kalman filter, it is necessary to determine the initial parameter values before their estimation itself. Taking into account the higher number of the parameters estimated in these extended models, more severe problems are likely to occur during the maximisation of the likelihood function, i.e. the optimisation of the filter. Instead of estimating the parameters within the model, the aforementioned issues are prevented by the calibration of the parameters and their estimation outside of the model.

The multivariate Kalman filter is becoming more and more popular among researchers and policy makers for the modelling of unobserved variables. The European Commission and OECD use the multivariate Kalman filter for the estimation of the NAIRU. The European Commission also utilises the multivariate Kalman filter in the estimation process of the trend in total factor productivity[3].

2.3 Structural methods

2.3.1 Production function with exogenous inputs

The most popular structural method for the estimation of the potential output is the production function method. The majority of applications consider the Cobb-Douglas production function:

$$Y_t = SPVF_t {N_t}^{\alpha} K_t^{\beta} \tag{12}$$

Where Y_t is the production output, $SPVF_t$ is the total factor productivity, N_t is the production factor of labour and K_t the capital stock. The parameters α and β represent the elasticities of output to labour and capital, respectively. Generally, the methodology adopts the assumption about the constant return to scale[4], where $\alpha + \beta = 1$, or $\beta = 1 - \alpha$. The production function may then be rewritten as:

$$Y_t = SPVF_t {N_t}^{\alpha} K_t^{1-\alpha} \tag{13}$$

To estimate potential output, the parameter α is often computed as the share of labour on the overall GDP. Total factor productivity is unobservable and it is thus obtained as the so called Solow residual from the logarithmic form of the production function:

$$spvf_t = y_t - \alpha n_t - (1-\alpha) k_t \tag{14}$$

Where the small cap letters denote the logarithms of the original series.

Potential output can then be derived by substituting the actual values of total factor productivity and the production factor of labour by their trends, $SPVF_t^*$ and N_t^* respectively:

$$Y_t^p = SPVF_t^* N_t^{*\alpha} K_t^{1-\alpha} \tag{15}$$

The trend in total factor productivity is often calculated using the Hodrick-Prescott filter, but there are other options as in the case of the direct estimation of potential output. For example, the European Commission uses the multivariate Kalman filter to estimate the trend total factor productivity with capacity utilisation as a supplementary indicator.

The trend in the production factor of labour is defined as:

$$N_t^* = hod_t^* pr_t^* pop_t (1 - u_t^*) \tag{16}$$

Where hod_t^* stands for the trend in the hours worked, pr_t^* for the trend labour market participation, pop_t^* is the population in productive age, and u_t^* is the equilibrium rate of unemployment, that is the already mentioned NAIRU. The last component of potential output is capital stock. Usually, its actual value enters the calculation as the maximum contribution to output is given by its full utilisation.

The trend values could be estimated by the fully defined structural model, but usually the production function is complemented by the exogenous inputs. In other words these are observed values of the time series entering the production function which have been adjusted using the statistical filters. The HP filter is predominantly used to extract the trend values from the time series of the hours worked, labour market participation, and unemployment. Lately, the

[3] A step in the application of the production function method for the estimation of potential output (section 2.3.1)

[4] Doubling both inputs of labour and capital will also double the overal output.

multivariate Kalman filter has become a popular method to estimate u_t^* considering Phillips curve which is modelled using price or wage inflation and potentially more exogenous variables.

The main criticism of the production function method lies in the preceding description of the construction of the potential output estimate. If the exogenous inputs enter the production function, it is necessary to derive their trends using the statistical filters, which suffer from the similar drawbacks as if applied directly on the time series of real output (Mc Morrow & Röger, 2001). Contrary to the Kalman filter, it is impossible to statistically evaluate the uncertainty about the resulting estimate. Additionally, the questioned issues include the Cobb-Douglas specification of the production function, the elasticities of output to labour and capital inputs as well as the total factor productivity. Lastly, the literature mentions inaccuracies in the estimation of the capital stock.

Nevertheless, the production function approach has some very positive features. Most importantly, it is possible to decompose the growth of potential output to contributions of individual components contained in the production function. Therefore, it is possible to distinguish to what extent growth of potential output is driven by the demographic changes, conditions of the labour market, changes in capital stock or the factor productivity. Making use of these calculations, one can illustrate for example the impact of the labour market reform on potential output growth. The relative weights of the labour and capital in the production can also be suited to the individual economy. In addition, the production function methodology does not suffer from the end-point bias characteristic to the HP filter. Although the HP filter is used to estimate the trend of inputs in the production function, these time series are usually much less volatile than the real output and therefore the estimates may be efficiently adjusted to minimise the end-point bias.

Production function methodology is the methodology used by the OECD, EC, and primarily also by the MF CR to estimate the potential output and the output gap. However, the methodologies differ in the construction of the trend time series of inputs and even in the specification of the production function. Compared to the aforementioned institutions, the MF CR works with the different weights of labour and capital and the trends are estimated using the HP filter. The EC on the other hand utilises the equilibrium unemployment rate and the trend total factor productivity estimated within the multivariate Kalman filters. A similar methodology is applied by the OECD, although it considers price inflation in the Phillips curve specification.

Table 2.2: Differences in production function methodology by EC and MF CR

	MF CR	EC
Methodological		
Parameters	parameter alpha calculated as a ratio between compensation of employees and the sum of compensation of employees and gross operating surplus (less mixed income)	fixed parameter α = 0.65
Output gap definition	with respect to the Gross Value Added (GVA*)	with respect to the GDP
Filters	HP filter (SPVF), moving averages	HP filter (output/capital ratio, hours worked and participation rate) Kalman filter (NAWRU** and SPVF)
Data inputs		
Unemployment	LFS	national accounts (take over data from LFS)
Participation rate	ratio of the number of employed people (disregarding the age group) to the economically active population 15-64	economically active population 15-74 (aligned with the approach of the AWG***)
Capital stock	values from the national accounts adjusted for the capacity utilization	national accounts
Hours worked	weekly hours worked, LFS	yearly hours worked, national accounts

*Note: * The methodological change from GDP to GVA based output gap is discussed in MF CR (2014a).*
*** NAWRU is the non-accelerating wage rate of unemployment. It is the equivalent of NAIRU, where wage inflation is used as an indicator instead of consumer price index (CPI).*
**** AWG is a European Policy Committee's (EPC) Working Group on Ageing Populations and Sustainability at the European Commission. The main focus of the Working Group is on long-term projections of social systems and sustainability assessment.*
Source: Havik et al. (2014) and MF CR.

2.3.2 Structural vector autoregression

The structural vector autoregression (SVAR) method for the estimation of potential output and the output gap builds upon the study by Blanchard & Quah (1989). The calculation relies on an assumption about economic shocks. For the decomposition of the actual output to the trend and cycle, the method uses the additional information about employment and capacity utilisation. As usual, the trend component is then interpreted as potential output and the cyclical component as the output gap.

In its commonly used form, the method assumes that employment, l_t, and capacity utilisation, cu_t, are affected by the same shocks as actual output y_t. Additionally, the time series are assumed to be stationary and they can be rewritten as a combination of contemporary and lagged structural shocks:

$$\begin{aligned}\Delta y_t &= \sum_{k=0}^{\infty} s_{11}(k)\eta_{1,t-k} + \sum_{k=0}^{\infty} s_{12}(k)\eta_{2,t-k} + \sum_{k=0}^{\infty} s_{13}(k)\eta_{3,t-k} \\ l_t &= \sum_{k=0}^{\infty} s_{21}(k)\eta_{1,t-k} + \sum_{k=0}^{\infty} s_{22}(k)\eta_{2,t-k} + \sum_{k=0}^{\infty} s_{23}(k)\eta_{3,t-k} \\ cu_t &= \sum_{k=0}^{\infty} s_{31}(k)\eta_{1,t-k} + \sum_{k=0}^{\infty} s_{32}(k)\eta_{2,t-k} + \sum_{k=0}^{\infty} s_{33}(k)\eta_{3,t-k}\end{aligned} \tag{17}$$

Where $\eta_{1,t}$, $\eta_{2,t}$ a $\eta_{3,t}$ are uncorrelated residuals, $\eta_{i,t} \sim N(0, \sigma_{\eta,i}^2)$, and $s_{ij}(k)$ are the corresponding coefficients. $\eta_{1,t}$ represents the supply shock, whereas $\eta_{2,t}$ a $\eta_{3,t}$ are considered demand shocks.

The estimation consists of several steps. First, the vector autoregression model is applied to the series of real GDP, employment, and capacity utilisation. In other words, every single variable is explained by its own lagged observations as well as by the lagged observation of the other variables. The residuals from this model are in essence a combination of demand and supply shocks. With the additional identification structure for the residuals, where the long-term growth is assumed to be inert to demand shocks, while supply shocks may exhibit a long-term influence, we can calculate potential output and the output gap as a sum of the preceding shocks:

$$\begin{aligned}\Delta y_t^p &= \sum_{k=0}^{\infty} s_{11}(k)\eta_{1,t-k} \\ \Delta y_t^c &= \sum_{k=0}^{\infty} s_{12}(k)\eta_{2,t-k} + \sum_{k=0}^{\infty} s_{13}(k)\eta_{3,t-k}\end{aligned} \tag{18}$$

Even though this method is advantageous by considering information from multiple time series and introduces assumptions about shocks of a various nature, structural vector autoregression is not used for the estimation of potential output very often. Cooley & Dwyer (1998) argue the reason is primarily the weak theoretical basis for shock identification structure, which can have a serious impact on the results. Mc Morrow & Röger (2001) also point out the high volatility of the estimated trend compared to the other methods.

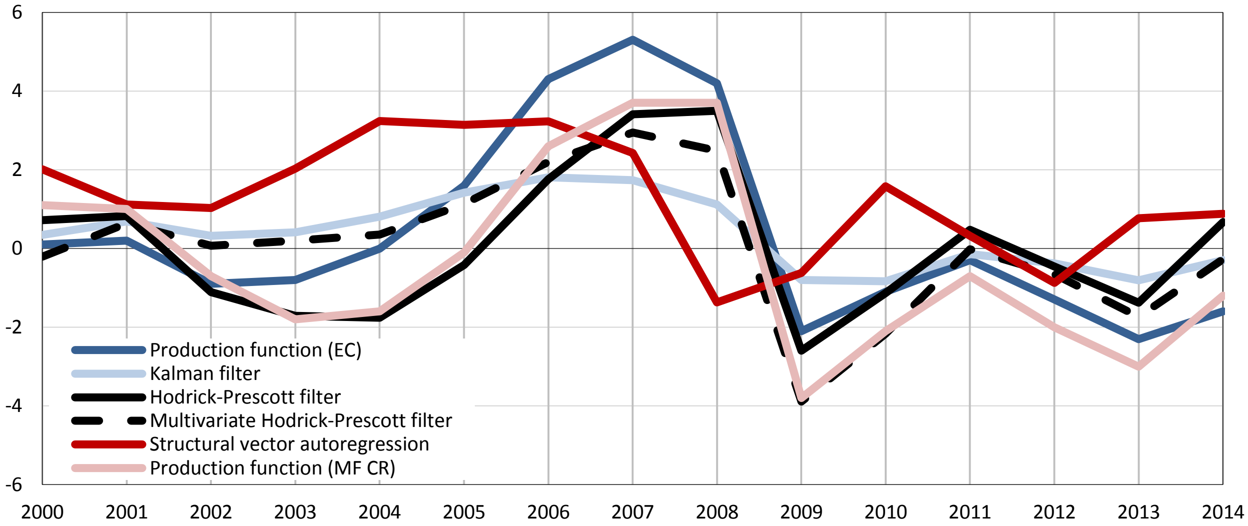
2.4 Comparison of different methods applied on the Czech data

Figure 2.5 presents the output gaps obtained by the application of different methods on the Czech data. Methodologies used for estimation include the Kalman filter in the basic specification, the production function method of the MF CR, and the production function method of the EC. The figure also displays the results of structural vector autoregression, where real output, employment and capacity utilisation were considered, as specified in the section 2.3.2. Last but not least, the results coming from Hodrick-Prescott filter in its univariate ($\lambda = 1600$) and the multivariate version are available for comparison. In the multivariate specification of the HP filter, Phillips curve, Okun's law, and capacity utilisation enter the estimation.

In general, we can conclude the methods provide similar information about the position of the economy within the business cycle. Still, the absolute values of the output gap differ given the method applied. The SVAR method is an exception to this rule. There the estimated cycle is shifted forward as a result of the higher sensitivity of the output to the other considered macroeconomic variables (see section 2.3.2).

Figure 2.5: Estimated output gap on the Czech data using different available methods

(in % of potential output)



Source: MF CR calculations.

Table 2.3: Comparison of methods to estimate potential output and the output gap – an overview

Methodology	Advantages	Disadvantages
Linear trend	easy implementation	potentially biased estimation of the potential output
HP filter	easy implementation symmetric decomoposition of the GDP time series	end-point bias arbitraty selection of λ parameter potential correlation of the estimated cycles
BK filter	frequency analysis lower risk of the cycle correlation	missing observations at the end of the sample
Kalman filter	detailed cycle definition parameter estimation	sensitivity to initial parameter values filter optimization
Multivariate HP filter	consideration of multiple time series	arbitrary selection of parameters complex estimation process
Multivariate Kalman filter	consideration of multiple time series based on relationship from economic theory	sensitivity to initial parameter values filter optimization
SVAR	including information from multiple time series	high volatility of estimated potential output identification of shocks
Production function	based on relationship from economic theory decomposition of potential output growth into individual factors	input series filtering estimating the capital stock

3 Cyclical adjustment of balance

Methods of the cyclical adjustment of balance can be divided into two main groups. The first group is based on the description of the state and development of the economy through potential output and output gap and the development of those items of government revenue and expenditure[5], which are sensitive to GDP development; we especially include methods developed and used by the EC, OECD and MF CR. The second group of methods is based on the relation of sensitive revenue and expenditure items outright to relevant macroeconomic bases. The most important representative of this group is the method of the ECB, which has also been applied by the MF CR.

Comprehensible interpretation stands as one of main advantages of the group of methods based on the description of cyclical revenue and expenditure development through the output gap. The dependence of the tax and social revenue and unemployment expenditure on the economic cycle is in accord with the economic theory.

Temporal desynchronisation of the cyclical development of individual cyclically sensitive items in years of considerable cyclical divergence is the main disadvantage of the first group of methods.

A more faithful description of the cyclical development of individual cyclically sensitive items of government revenue and expenditure is the main advantage of the ECB method. Their cyclical development is not always completely synchronised with the cyclical development of the GDP; and so, we can describe the cyclical and cyclically adjusted components of government revenue and expenditure items more precisely through the use of specific macroeconomic bases.

Relatively large sensitivity of the result to discretionary changes of cyclically sensitive revenue and expenditure items is one of the disadvantages of the ECB method. In the environment of frequent discretionary changes of these items (e.g. because of legislative reasons), estimates of the elasticities of sensitive items to macroeconomic bases tend to be less reliable. Limits stemming from the choice of the HP filter for the calculation of the potential values of macroeconomic bases are another disadvantage of the ECB method.

3.1 Methods based on the output gap of the economy

3.1.1 OECD Method

The family of methods using the elasticity of sensitive items of government revenue and expenditure to the output gaps is based on the Van den Noord (2000) paper by the OECD and further developed in Girouard and André (2005) and Price *et al.* (2014).

The decomposition of the government balance is summarised by the following equation:

$$B = CAB + CB \tag{19}$$

Here, the balance of government sector *B* equals the sum of the cyclically adjusted balance *CAB* and the cyclical component of balance *CB*. The cyclically adjusted balance of the government sector is therefore the difference of the cyclically adjusted revenue and expenditure, to which the remaining cyclically insensitive component of balance is added. In ratio to the potential output ($cab = CAB/Y^p$) thus:

$$cab = \frac{\sum_i R_i^* - G^* + X}{Y^p} \tag{20}$$

Where R_i* is the cyclically adjusted component of the *i-th* category of the budgetary revenue, *G** is cyclically adjusted government expenditure, *X* stands for the remaining items of the balance which are not sensitive to the economic cyclical development and Y^p is the potential output.

We will obtain cyclically adjusted components of revenue and expenditure by the following calculation:

$$R_i^* = R_i \left(\frac{Y^p}{Y}\right)^{\eta_{R_i}} \tag{21}$$

$$G^* = G \left(\frac{Y^p}{Y}\right)^{\eta_G}$$

[5] MF CR (2013) provides the definition of the government sector and other concepts in the area of public finance.

Where R_i is the value of the *i-th* category of revenue, G^* is the value of government expenditure, Y is the output (gross domestic product), η_{Ri} is the elasticity of the *i-th* category of revenue to the output gap and η_G is the elasticity of government revenue to the output gap.

By the substitution of the relations (21) into the equation (20) and by additional algebraic adjustments based on the decomposition of the balance in the equation (19), we obtain a cyclical component of balance in ratio to the potential output:

$$cb = \frac{1}{Y}\sum_i R_i\left[1-\left(\frac{Y^p}{Y}\right)^{\eta_{R_i}-1}\right]-\frac{G}{Y}\left[1-\left(\frac{Y^p}{Y}\right)^{\eta_G-1}\right]+\frac{X}{Y}\left[1-\left(\frac{Y^p}{Y}\right)^{-1}\right] \tag{22}$$

Revenue sensitive to the economic cycle can be divided into four categories according to the OECD method: corporate income tax, personal income tax, social security contributions (social security and healthcare contributions in the Czech Republic) and indirect taxes. Unemployment benefits are the only cyclically sensitive expenditure according to the OECD method.

The estimation of elasticities of cyclically dependent items is a two-step procedure. The estimation of the sensitivity of tax revenue (respectively benefit expenditure) to the change in the tax base (respectively unemployment) is the first step. The estimation of sensitivity of the base to the cycle measured by the output gap is the second step.

The balance can also be adjusted, aside from the economic cycle, by the effect of other factors, which in some cases can be significant:

- by the price of assets, as described in detail in Price and Dang (2011), taking into account the price development of shares and immovables,
- for economies with a large ratio of revenue from raw material (oil, copper, molybdenum, etc.) extraction to the GDP, it is possible to adjust the balance by the impact of the change in the price of these commodities on the world market (see Baunsgaard *et al.*, 2012),
- by the non-proportional development of the expenditure components of the GDP, described in detail in Bornhorst *et al.* (2011),
- by the further adjustment of the cyclically adjusted balance of one-off revenue and expenditure, we obtain the structural balance.

3.1.2 The European Commission method

The EC is using the modified method of the OECD, which it actualises in certain time intervals, to adjust the government sector balance of the impact of the economic cycle. The current update of the method was performed in 2012–2014 in cooperation with the OECD, which is described in detail in Mourre *et al.* (2014). Apart from replacing the sensitivities by semi-elasticities, the individual elasticities of revenue items have been re-estimated; for expenditure, elasticities were also estimated for other social benefits than just unemployment benefits, but the assumption of the cyclical behaviour of the other benefits was rejected by statistical tests.

The cyclically adjusted balance in relative terms (*cab*) is the difference of the cyclically adjusted revenue and expenditure in ratio to the potential output:

$$cab_t = \frac{B_t^*}{Y_t^p} = \frac{R_t^* - G_t^*}{Y_t^p} \tag{23}$$

The last figure in equation (23) can be transformed to:

$$\frac{R_t^* - G_t^*}{Y_t^p} = \frac{R_t}{Y_t^p}\left(\frac{R_t^*}{R_t}\right) - \frac{G_t}{Y_t^p}\left(\frac{G_t^*}{G_t}\right) \tag{24}$$

Thus the cyclically adjusted balance can be, after substituting for the ratio of cyclically adjusted revenue to total revenue and the ratio of cyclically adjusted expenditure to total expenditure in equation (21), expressed as:

$$cab_t = \left(\frac{R_t}{Y_t^p}\right)\left(\frac{Y_t^p}{Y_t}\right)^{\eta_R} - \left(\frac{G_t}{Y_t^p}\right)\left(\frac{Y_t^p}{Y_t}\right)^{\eta_G} \tag{25}$$

With respect to the definition of the relative output gap as $OG_t = \left(Y_t - Y_t^p\right)/Y_t^p$, it is possible to express the potential output in relation to the output gap as $Y_t^p = Y_t/(1 + OG_t)$ and the ratio of the potential output to the real output therefore as $Y_t^p/Y_t = (1 + OG_t)^{-1}$ so the cab_t formula can be further rewritten as:

$$cab_t = \frac{R_t}{Y_t}(1 + OG_t)^{1-\eta_R} - \frac{G_t}{Y_t}(1 + OG_t)^{1-\eta_G} \tag{26}$$

Which can be linearly approximated in the form:

$$cab_t \cong \frac{R_t}{Y_t}(1 + (1 - \eta_R)OG_t) - \frac{G_t}{Y_t}(1 + (1 - \eta_G)OG_t) \tag{27}$$

And further rewritten as:

$$cab_t \cong \frac{R_t}{Y_t} - \frac{G_t}{Y_t} - \left(\frac{R_t}{Y_t}(\eta_R - 1) - \frac{G_t}{Y_t}(\eta_G - 1)\right)OG_t = \frac{B_t}{Y_t} - (\varepsilon_R - \varepsilon_G)OG_t = \frac{B_t}{Y_t} - \varepsilon \cdot OG_t \tag{28}$$

The final form on the right side of the equation (28) has been chosen by the EC for the direct application on the relative output gap for the cyclical adjustment of the government balance:

$$cab_t = \frac{B_t}{Y_t} - \varepsilon \cdot OG_t \tag{29}$$

The overall semi-elasticity ε is expressed through the elasticities η_{Ri} of the particular *i*-th revenue item: personal income tax, corporate income tax, indirect taxes (value added tax and excises), social security contributions and non-tax revenue[6] to the output gap and weighted by the ratio of this revenue to total revenue; and through the elasticity η_{Gu} of the unemployment benefits to the output gap weighted by the ratio of unemployment benefits to total expenditure:

$$\varepsilon = \frac{d\left(\frac{B}{Y}\right)}{\frac{dY}{Y}} = \left(\sum_{i=1}^{5} \eta_{R_i}\frac{R_i}{R} - 1\right)\frac{R}{Y} - \left(\eta_{G_u}\frac{G_u}{G} - 1\right)\frac{G}{Y} \tag{30}$$

The weights of individual revenue and expenditure items and the ratios of revenue and expenditure to the GDP have been set as the average of the years 2002–2011. Further actualisations of the weights should take place after six years, the next one thus in 2020. New estimates of individual elasticities are proposed by the EC to be calculated after ca. 9–10 years. The value of the overall semi-elasticity is fixed in time and will not be actualised till the new weight or elasticity estimates. Based on actual EC estimates, the value of the overall semi-elasticity of government revenue and expenditure to the output gap is $\varepsilon = 0.43$ for the Czech Republic.

Individual revenue and expenditure elasticities have been, similarly as in the original OECD method, calculated in two steps, which are the calculation of the elasticity of the revenue or expenditure item to the relevant base and the calculation of the elasticity of the base to the output gap. Table 3.1 shows the summary of elasticities of revenue and expenditure items and the relevant bases[7] for the Czech Republic.

Table 3.1: Elasticities of cyclical sensitive revenue and expenditure in the Czech Republic

Revenue or expenditure item	Base	Elasticity of item to base	Elasticity of base to output gap	Elasticity of item to output gap
Personal income tax	Weighted average of bases for taxes	2.23	0.74	1.65
Corporal income tax	Net operating surplus	1.23	1.45	1.78
Social security contributions	Compensations of employees	0.99	0.87	0.86
Indirect taxes	Consumption of households	1.00	1.00	1.00
Unemployment benefits	Rate of unemployment	1.00	-2.45	-2.45

Source: Mourre et al. (2014). Adjusted by MF CR.

The results of the estimates of indirect tax elasticities were not statistically significant for a large part of EU countries, therefore the hypothesis of unitary elasticity has been adopted. The unitary elasticity hypothesis of unemployment benefits to unemployment has been adopted a priori. Statistical tests did not confirm the cyclical behaviour of non-tax revenue and other expenditure than unemployment benefits.

[6] The non-tax revenue elasticity has been estimated as zero.

[7] In the case of personal income tax, the overall base is the weighted average of particular bases, which are the compensation of employees, the income of the self-employed and capital income.

3.1.3 Methods used by the Ministry of Finance

The Ministry of Finance of the Czech Republic has been consistently using the OECD method for the calculation of the cyclically adjusted balance, published in regular publications which are the Convergence Programme of the Czech Republic, the Macroeconomic Forecast of the Czech Republic and the Fiscal Outlook of the Czech Republic.

For the output gap calculation, the MF CR uses the production function method described in 2.3.1. For the determination of the cyclical component, the actual EC elasticities estimates, which are shown in Table 3.1, have been used since the autumn of 2014. The results of the revisions of elasticities were not much different for the Czech Republic from previous values, detailed information can be found at the MF CR (2014b). However, the MF CR does not, unlike the EC, assume the value of the overall elasticity of government revenue and expenditure to the output gap to be fixed in time; the MF CR uses the actual weights of cyclically sensitive government revenue and expenditure for each year. This approach is suitable in the Czech environment because of the more flexible representation of relatively frequent changes of the tax code and its impact on the composition of government revenue. The MF CR uses, for the calculation of cyclically adjusted components of government revenue and expenditure, the definition of cyclical components in a non- linear form, i.e. according to the equation (21).

3.2 The European Central Bank method

Since 2013, the Ministry of Finance has been publicising the estimates of the cyclical component of the government balance also by the ECB method for comparison in the Fiscal Outlook of the Czech Republic. This method, presented in the paper Bouthevillain *et al.* (2001), aims to take into the account the impact of a different cyclical development of particular items of government revenue and expenditure in time. For this reason, it is not based on the relation of revenue and expenditure to the cyclical development of the common macroeconomic indicator (GDP) described by the output gap, but on the cyclical development of particular macroeconomic bases relevant for each item of cyclically sensitive revenue and expenditure.

As in the case of methods based on the relation of sensitive items to the output gap, even in the ECB method the process of cyclical adjustment takes two steps. At first, the cyclical position of the economy is evaluated through the comparison of real values of macroeconomic bases to their potential values, which are calculated by the Hodrick-Prescott filter (see 2.1.2). The calculation of the impact of the development of macroeconomic bases on government revenue and expenditure (i.e. of their cyclical component) through the estimate of elasticities of real revenue and expenditure to macroeconomic bases is the second step. We thus obtain the cyclically adjusted balance as the difference between the actual value of the government balance and the cyclical component calculated in that manner.

The relation between government revenue or an expenditure item and the relevant macroeconomic base is described by the following equation:

$$\Delta \ln B_t^j = \alpha + \delta t + \beta \Delta \ln V_t^j + A_t + \xi_t \tag{31}$$

Where B^j is the revenue or expenditure item, V^j is the relevant macroeconomic base, the parameter α is related to the trend change in fiscal ratios, the parameter β measures the elasticity of the revenue or expenditure item to the macroeconomic base. The variable A captures the impact of discretionary fiscal policy measures, respectively the impact of idiosyncratic events (in the Czech environment e.g. floods). The parameter δ captures the change of the composition of sensitive items in time, but for the Czech Republic, statistical tests by the MF CR did not confirm the statistical significance of this parameter. The parameter $\xi \to 0$ is the error.

The second alternative ECB equation for the elasticity calculation uses time-lagged variables in the error correction model:

$$\Delta \ln B_t^j = \alpha + \beta\left(\ln B_{t-1}^j - \gamma \ln V_{t-1}^j + \phi + \delta t\right) + \delta_1 \Delta \ln V_t^j + \delta_2 \Delta \ln V_{t-1}^j + A_t + \xi_t \tag{32}$$

The parameter γ measures the long-term relationship, and the parameters δ_1 and δ_2 refer to the short term development of elasticities. As we used annual data for our calculation, we refrained from use of this approach modelling the short-term development of elasticities.

Of course, it is necessary to make input series stationary for estimating elasticities. On account of that, the year-on-year difference in real terms in the logarithms of the fiscal and macroeconomic indicators in question has been used.

The summary of cyclically sensitive government revenue and expenditure in the Czech Republic, relevant macroeconomic bases and estimates of elasticities of items to bases is shown in Table 3.2. Elasticities of corporate income tax, of social security contributions and of unemployment benefits were estimated in accordance with equation (31). As the results of the estimates of elasticities of indirect taxes and personal income tax were not statistically significant, the assumption of unitary elasticity of these taxes to their bases has been accepted.

Table 3.2: Elasticities of revenue or expenditure items to macroeconomic bases according to the ECB method

Revenue or Expenditure Item	Macroeconomic Base	Elasticity
Value Added Tax	**Private Consumption Expenditure**	**1.00**
Personal Income Tax	**Wage Bill in the Private Sector**	**1.00**
Corporate Income Tax	**Net Operating Surplus**	**1.02**
Social Security Contributions	**Wage Bill in the Private Sector**	**0.49**
Unemployment Benefits	**Unemployment** (according to the Ministry of Labour and Social Affairs)	**1.30**

Source: MF CR calculations.

The Hodrick-Prescott filter with parameter λ = 30 was used to calculate the potential values of macroeconomic bases. The relatively low value of the parameter was chosen with respect to the greater robustness to structural breaks in the time series trends, which is methodologically appropriate for the Czech Republic not only as for an economy in the post-transformation phase, but also for the correct representation of the impact of the still recent world economic crisis. The relative gap of the macroeconomic base is calculated as follows:

$$v_c^j = \frac{V^j - V^{j*}}{V^{j*}} \qquad (33)$$

Where V^j is the real value of the macroeconomic base, V^{j*} is the trend value obtained by the application of the HP filter.

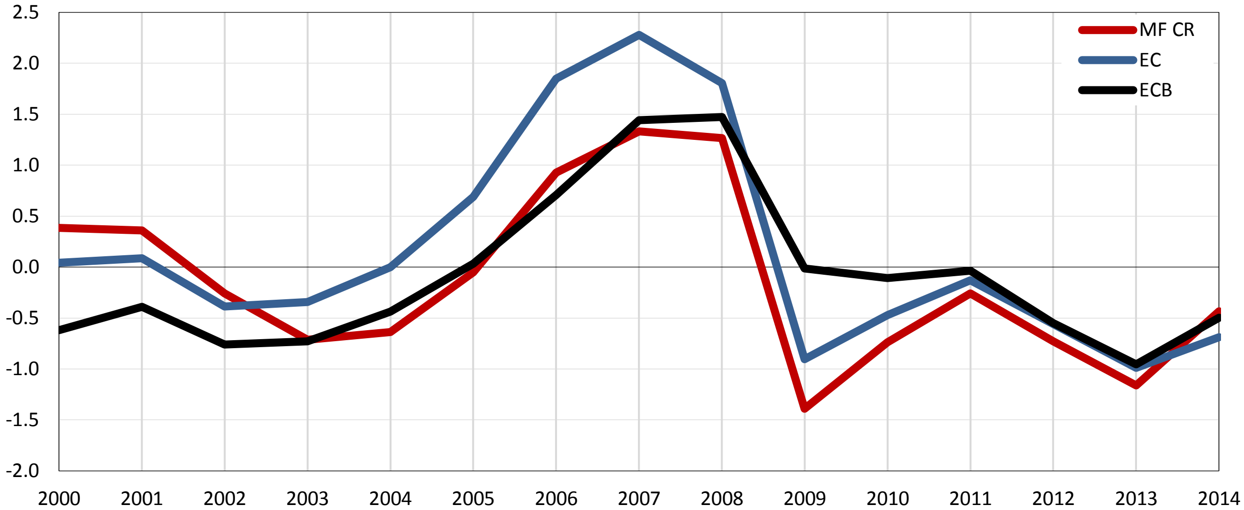
The cyclical component of individual government revenue and expenditure items in current prices is calculated as the product of value of the item B^j in current prices, of the elasticity β to the relevant base and of the relative gap v_c^j of the macroeconomic base:

$$B_c^j = B^j \cdot \beta_{B^j,V^j} \cdot v_c^j \qquad (34)$$

3.3 Comparison of the results of the cyclical adjustment of balance

The figures below depict the comparison of specific methods of the cyclical adjustment of balance. First, the method used by the MF CR, which uses the Cobb-Douglas production function (see subchapter 2.3.1) for the estimate of the potential output and the method of OECD is shown, which applicates partial elasticities on individual items of revenue and expenditure instead of overall semi-elasticity (see 3.1.1), is used for the cyclical adjustment. To this method, we have added comparison with the applied method of the ECB, which, in contrast with other methods, relates elasticities to gaps of specific macroeconomic bases of revenue and expenditure. Of course, the method of the EC, which applies the overall semi-elasticity of revenue and expenditure to the output gap calculated by the Cobb-Douglas production function, is also included. The comparison is shown in Figure 3.1.

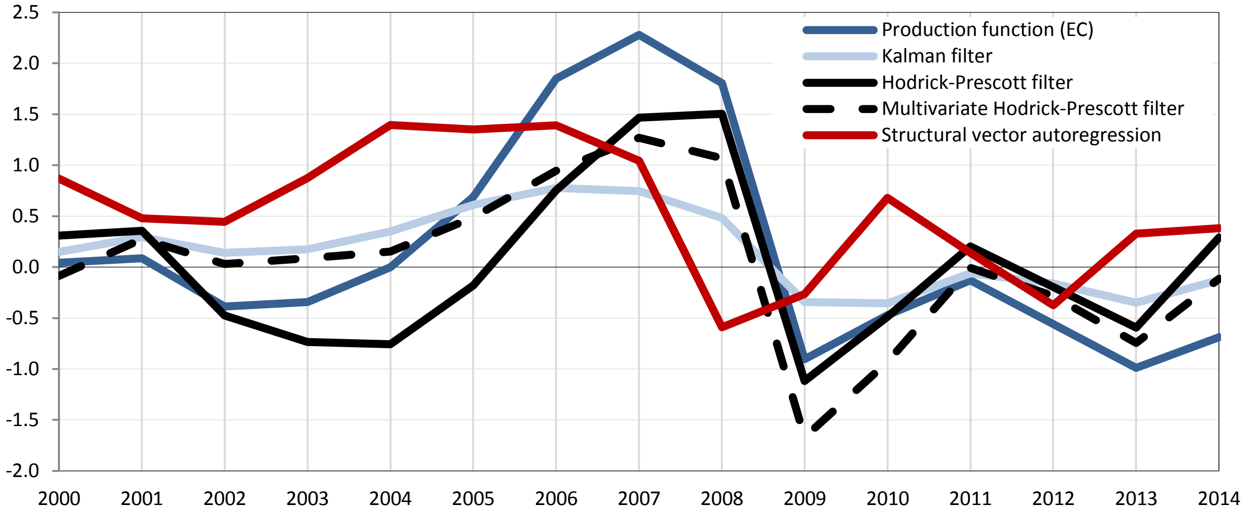
Figure 3.1: Cyclical component of balance according to MF CR, EC a ECB methods
(in % of GDP)



Source: MF CR calculations.

In order to demonstrate the impact of different estimates of the output gap on the cyclical component of balance, we have applied the overall semi-elasticity, used in the cyclical adjustment by the EC method, to results of different methods of the potential output calculation (uni- and the multivariate HP filter, the Kalman filter, structural vector autoregression) described in Chapter 2. Cyclical components thus obtained are shown in Figure 3.2. A numerical comparison of the value of the cyclical component and the structural balance calculated by the given methods is summarised in Table 6.1 in the Annex.

Figure 3.2: Cyclical component of balance according to different output gap calculation methods
(in % of GDP)



Source: MF CR calculations.

As is shown in both graphs, different methods provide different results, especially in years of large cyclical shifts. The EC method, which uses the production function for the output gap calculation and which assumes the fixed elasticities of sensitive items to the output gap, can lead to a certain stiffness in the time of marked cyclical shifts and can also overestimate the size of the positive output gap in the past. Both methods currently used by the MF CR, i.e. the OECD method (use of the production function and variable weights of sensitive items) and the ECB method seem to be quite appropriate for the calculation of the cyclical component of balance. The advantage of the OECD method is, in comparison with the presumption of fixed elasticities, a more flexible reaction to changes in the economic growth or the tax code. The main advantage of the ECB method is the more detailed depiction of the cyclical development of the economy through the use of specific partial macroeconomic bases. The disadvantage of the ECB method is primarily the higher demand of the input data quality, while statistical tests of certain variables did not confirm the significance of the relation between the time series of cyclically sensitive government revenue or the expenditure item and relevant macroeconomic base, and thus the expert estimate had to be accepted as the value of their elasticity. Another, rather formal disadvantage of the ECB method is that the European Central Bank is the only relevant international institution primarily using it; while the EC and OECD use methods based on the calculation of the potential output and the output gap. It should be noted that an expert discussion over the specification of methods of the cyclical adjustment of balance in working groups of the EC is currently underway.

From the comparison of the application of methods of the output gap calculation using the cyclical adjustment by the EC method, values obtained by the calculation of the output gap through the structural vector autoregression diverge the most from the average.

4 Conclusion

A plethora of methods exist for the estimation of potential output, the output gap, and cyclical adjustment of the government balance. The aim of this study was to provide their overview, map the potential differences in application on the Czech data, and also to briefly document the methods currently applied by the MF CR. The assessment of the position within the business cycle is crucial to pursue suitable economic policies and for the policy maker's decision making. Regarding the stabilisation function of the fiscal policy, it is important to limit its procyclical effects. Therefore, it is necessary that the applied methodology closely reflects these requirements.

The main finding of the study is that individual approaches to cyclical government balance adjustment provide similar conclusions about the business cycle, i.e. they are in accord in determining whether the economy is working above or below its productive capabilities. The exception to this is the SVAR method, where the high volatility of the estimated potential output reflecting the changes in the remaining variables – employment and capacity utilisation, shifts the estimated cycle. These results subsequently affect the calculations of the cyclically adjusted government balance, where the output gap from SVAR is employed. Future research should be devoted to the comparison of the methods with respect to their sensitivity to the input data revisions and their overall stability.

The MF CR currently uses the production function method to estimate potential output and the output gap. This methodology is in various adjusted forms also used in the majority of the EU countries. Its undisputed advantage is, apart from the straightforward international comparison of the business cycles, mainly the foundation in economic theory. The results therefore allow for an easier economic interpretation and provide an insight into the factors of potential output movements. Moreover, they also provide guidance for the policy makers as to what policies might be introduced to boost potential output growth. As far as the cyclical adjustment of the government balance is concerned, the methodology of the EC is the most frequently used in the EU. The main reason is probably the general agreement on this method by the Member States and its utilisation for the evaluation of the economic and fiscal development in the Member States by the EC. Other arguments in favour of this methodology include the low data and technical requirements as the methodology may be easily applied when the output gap of the economy is known.

According to the legislative proposal and the corresponding constitutional act about the budget responsibility, currently discussed by the Parliament of the Czech Republic, an independent fiscal institution National Budgetary Committee will be established. This institution should cooperate with the MF CR and propose a methodological approach to the calculation of the structural government balance. The cyclical adjustment of the government balance is an intermediate step towards computing the structural government balance and we hope this overview study will initiate an expert discussion on this topic.

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6 Table annex

Table 6.1: Cyclical component of balance and structural balance

	2008	2009	2010	2011	2012	2013	2014
% of GDP							
Cyclical component of balance according to the OECD method	**1.3**	**-1.4**	**-0.7**	**-0.3**	**-0.7**	**-1.2**	**-0.4**
Cyclical component of balance according to the ECB method	**1.5**	**0.0**	**-0.1**	**0.0**	**-0.5**	**-1.0**	**-0.5**
Cyclical component of balance according to the EC method							
Method of the output gap calculation:							
Production function according to the EC method	**1.8**	**-0.9**	**-0.5**	**-0.1**	**-0.6**	**-1.0**	**-0.7**
Kalman filter	**0.5**	**-0.3**	**-0.4**	**-0.1**	**-0.2**	**-0.3**	**-0.1**
Structural vector autoregression	**-0.6**	**-0.3**	**0.7**	**0.1**	**-0.4**	**0.3**	**0.4**
Hodrick-Prescott filter	**1.5**	**-1.1**	**-0.5**	**0.2**	**-0.2**	**-0.6**	**0.3**
Multivariate Hodrick-Prescott filter (MVHP)	**1.1**	**-1.7**	**-0.9**	**0.0**	**-0.3**	**-0.7**	**-0.1**
% of GDP							
Structural balance according to the OECD method	**-3.3**	**-4.2**	**-3.7**	**-2.3**	**-1.2**	**0.1**	**-1.4**
Structural balance according to the ECB method	**-3.5**	**-5.5**	**-4.3**	**-2.6**	**-1.4**	**-0.1**	**-1.3**
Structural balance according to the EC method							
Method of the output gap calculation:							
Production function according to the EC method	**-3.8**	**-4.7**	**-4.0**	**-2.5**	**-1.4**	**-0.1**	**-1.1**
Kalman filter	**-2.5**	**-5.2**	**-4.1**	**-2.5**	**-1.8**	**-0.7**	**-1.7**
Structural vector autoregression	**-1.4**	**-5.3**	**-5.1**	**-2.7**	**-1.6**	**-1.4**	**-2.2**
Hodrick-Prescott filter	**-3.5**	**-4.4**	**-4.0**	**-2.8**	**-1.7**	**-0.5**	**-2.1**
Multivariate Hodrick-Prescott filter (MVHP)	**-3.1**	**-3.9**	**-3.5**	**-2.6**	**-1.6**	**-0.3**	**-1.7**
CZK bn.							
Structural balance according to the OECD method	**-132**	**-164**	**-147**	**-94**	**-49**	**3**	**-59**
Structural balance according to the ECB method	**-140**	**-218**	**-172**	**-103**	**-56**	**-5**	**-57**
Structural balance according to the EC method							
Method of the output gap calculation:							
Production function according to the EC method	**-154**	**-183**	**-157**	**-99**	**-55**	**-4**	**-49**
Kalman filter	**-101**	**-205**	**-162**	**-102**	**-71**	**-30**	**-72**
Structural vector autoregression	**-58**	**-208**	**-203**	**-110**	**-63**	**-58**	**-94**
Hodrick-Prescott filter	**-142**	**-174**	**-156**	**-113**	**-70**	**-20**	**-90**
Multivariate Hodrick-Prescott filter (MVHP)	**-124**	**-153**	**-139**	**-104**	**-67**	**-14**	**-73**

Source: MF CR calculations.

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