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HUBERT: A DSGE MODEL OF THE CZECH REPUBLIC

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a DSGE model of the Czech economy

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Abstract

In the paper we present a simple DSGE model of the Czech economy called HUBERT. The model describes the behaviour of four basic agents in the economy: households, firms, government, and world. Although HUBERT is rather a simple version of standard DSGE models, it incorporates standard features of New Keynesian economics such as imperfect competition, habit formation of households, nominal and real rigidities. A current version of the model is intended both for policy analysis simulations and regular forecasts at the Ministry of Finance. From preliminary results follow that the model produce reasonable outputs.

Abstrakt

Ve studii prezentujeme jednoduchý DSGE model české ekonomiky s názvem HUBERT. Model popisuje chování čtyř základních subjektů v ekonomice: domácností, firem, vlády a vnějšího prostředí. I přesto že HUBERT je spíše jednoduchou verzí standardního DSGE modelu, obsahuje základní atributy nové keynesiánské ekonomie, jako nedokonalou konkurenci, spotřební zvyklosti domácností (*"habit formation"*), nominální a reálné rigidity. Současná verze modelu je určena jak pro simulaci, tak i pro pravidelné makroekonomické predikce prováděné na ministerstvu financí. Z pedběžných výsledků vyplývá, že model přináší rozumné výsledky.

Keywords: New Keynesian macroeconomics, dynamic stochastic general equilibrium model, solution of a DSGE model, calibration of DSGE model, impulse response functions.

JEL classification: E52, F31, F41.

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Contents

1 Introduction

1.1 Last development in macroeconomic modelling

The last 30 years have witnessed significant improvements in macroeconomic modelling. The most important changes consist of a size and a theoretical background of the models: from ad hoc specified models based on an IS-LM-BP approach through error-correction models[1] to currently used dynamic stochastic general equilibrium models.[2] DSGE models combine features of the RBC theory and New Keynesian Macoreconomics: optimization of rational agents extended with imperfect competition, habit formation, nominal and real rigidities. Rigidities in the model are introduced through indexation and/or costs of adjustments which are at the same time very useful in deriving a dynamic structure of key endogenous variables. Although DSGE models definitely represent significant improvements compared to traditional ad-hoc specified models, DSGE models still suffer from many shortcomings and some questions remain open, unfortunately.

1.2 Introduction to Hubert

The model describes the behaviour of four basic agents in the economy. The government regulates tax and transfer policy. The central bank, operating under the inflation targeting regime, determines a short term policy rate. Households supply labour to firms and spend their income for consumption. Business sector in the model is represented by three different firms: (i) importers buying goods for consumption and/or further production; (ii) producers combining imported and domestic inputs to produce intermediate products for retailers; (iii) retailers selling final goods to domestic agents (households and/or government) and/or export abroad. Since the Czech economy is a small open and raw materials dependent economy, foreign trade (mainly with other EU countries) is of the crucial importance. An illustrative overview of the main blocks of the model can be found in Figure 1.[3]

1.3 Introduction to model documentation

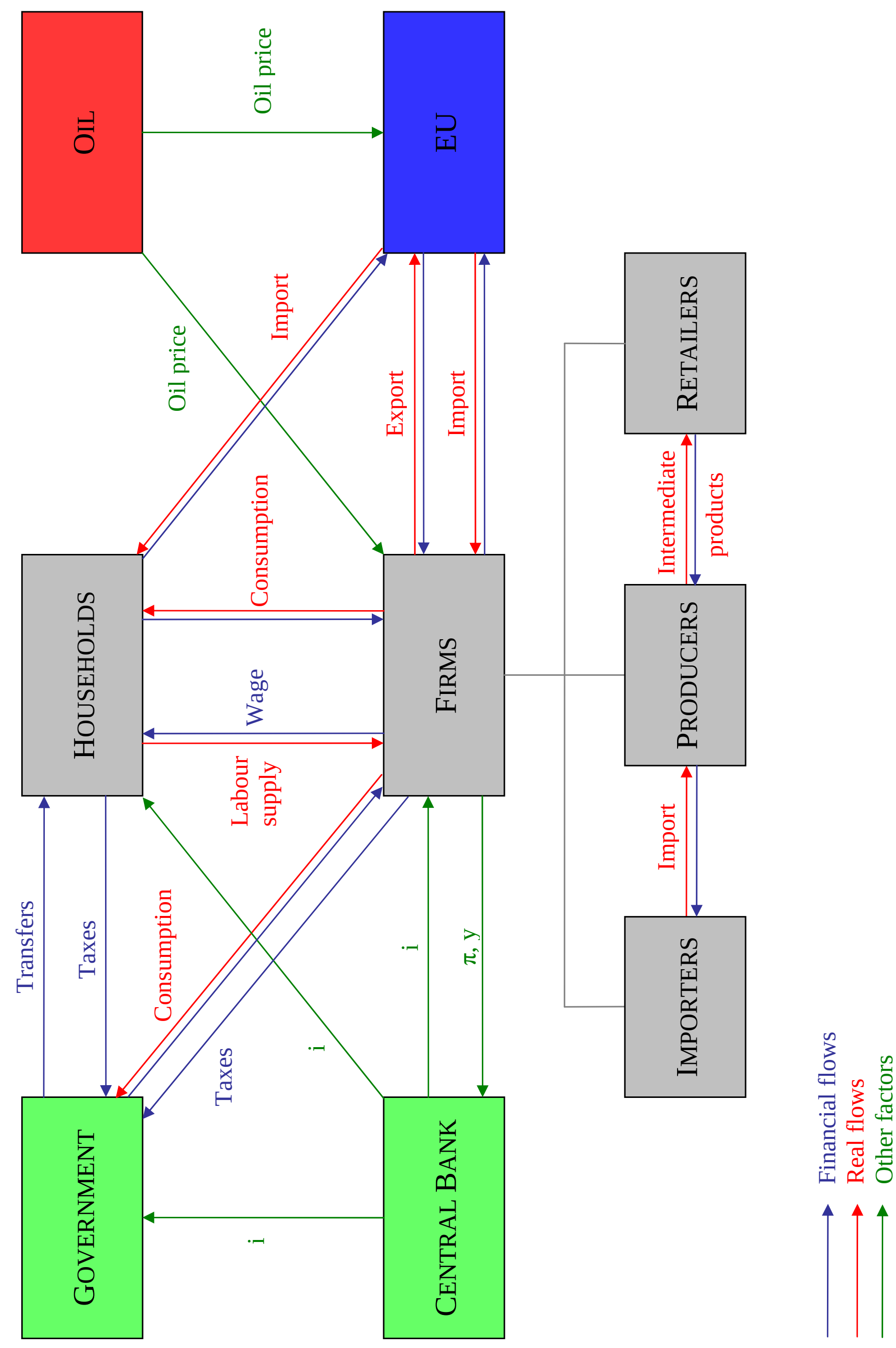
For a clear understanding of the model documentation we present a basic concept of data transformation. A variable denoted by a capital letter (e.g. X_t) is a variable in levels (thousands of employees, billions of CZK, indices, etc.). A variable denoted by a lower case letter (e.g. x_t) is natural logarithm of the original variable $x_t \equiv \ln X_t$ or some ratio in percentage (unemployment rate, government debt as a percentage of GDP, etc.).

[1] Prototypes of this class of models are: Canadian QPM, Japanese JEM, New Zealand FPS, Dutch JADE, Finnish EDGE or French MARCOS.

[2] Standard DSGE models are as follows: American SIGMA or European NAWM.

[3] Besides our model, another DSGE of the Czech economy has been developed by the Czech National Bank recently, see Beneš et al. (2005) for more information.

Figure 1: A bird's eye view of the model



2 A structure of the model

2.1 Households

We assume that a domestic economy is populated by infinitely lived and liquidity unconstrained consumers with a free access to the financial market (indexed by $i \in [0,1]$). We also assume that households maximize an intertemporal utility function subject to a lifetime budget constrain.[4]

The utility function is positively affected by consumption C_{it} and negatively by labour supply N_{it}. Specific forms of objective functions in the RBC/DSGE literature seem to be somewhat arbitrary but they are usually chosen with respect to some theory and/or empirical findings. In the case of the households' optimization problem we have to assume that an instantaneous utility function is concave and a budget constrain is a convex set in order to achieve a unique solution of the problem. However, there are many functions satisfying this condition. Therefore, we have to impose additional restrictions on households' preferences. For example, we know from business cycle facts that consumption and output exhibit approximately the same constant growth rate over time which implies that consumption-output ratio is approximately constant over time. From this reason, we should specify the utility function of households from the admissible set of functions ensuring a balanced growth path of the model. King et al. (1988) proofed that the constant relative risk aversion function (CRRA) with parameter $\psi_c = 1$ satisfy all necessary conditions. Moreover, this function is concave, continuous and easy to differentiate.

We also incorporate habit formation[5] according to Abel (1989) and Fuhrer (2000).

[4]Only for simplicity we assume the existence of Ricardian households who are liquidity unconstrained with a free access to the financial market in order to smooth their consumption. Although this simplifying assumption is far away form realism and therefore very often criticized, Evans (1991) showed that the Ricardian equivalence concept can still be quite a good and useful approximation. Nevertheless, the quality of the approximation is determined by many specific factors such as financial market imperfections, uncertainty, intergenerational transfers, etc. Moreover, Evans (1993) tested a hypothesis about Ricardian equivalence against Blanchard (1985) model using a sample of 19 developed countries. The main finding is that the null hypothesis about Ricardian equivalence cannot be rejected for 18 of 19 countries in a given sample. So, the specification of households in our model follows the above mentioned results. However, there are two good reasons for incorporating some "rule of thumb" households into the model: a) the Czech economy is not comparable with the most developed countries in the sample and therefore the results can be rough approximations; b) incorporating liquidity constrained households can have serious implications on the whole model dynamics and some other simulation results. Rule-of-thumb households will be incorporated into the next version of the model.

[5]Although we found some weak evidence in favour of the habit formation from empirical data sets, there is virtually no single test for distinguishing between different types of habits (external, internal and deep habits). Hence, the specification of habit formation is fully up to model builders. We use the external additive form of habits from two reasons: a) simplicity of the derived FOCs; b) the relative risk parameter can be a constant. This feature does not have to hold for other types of habits.

Habit formation of consumption is then defined as $H_t^c = \gamma C_{t-1}$, where γ represents the habit persistence parameter, measuring the effect of past consumption ($0 \leq \gamma \leq 1$). Moreover, following the arguments in Lettau and Uhlig (2000), habit formation in labour supply is not incorporated in to the model since it can have an adverse effect on model dynamics. Thus an optimization problem concentrates in maximization of the following utility function

$$\max_{\{C_{it}, A_{it}, A_{it}^*, N_{it}\}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{it} - H_t^c)^{1-\psi_c}}{1-\psi_c} - \frac{N_{it}^{1+\psi_n}}{1+\psi_n} \right], \tag{1}$$

where:

A_{it}, A_{it}^*	bunch of net domestic and net foreign assets held by a household,
C_{it}	individual real consumption of household,
H_t^c	habit level of consumption,
N_{it}	individual labour supply,
β	discount factor,
ψ_c	inverse of substitution of consumption,
ψ_n	inverse of substitution of labour supply.

As mentioned earlier, households are assumed to maximize the utility function subject to a budget constraint given by

$$\begin{aligned} A_{it} + S_t A_{it}^* + (1+\tau_c) P_t C_{it} = {} & (1+i_{t-1}) A_{it-1} + (1 + i_{t-1}^* + \zeta_{t-1}^*) S_t A_{it-1}^* + \\ & + (1 - \tau_w + \tau_b) W_{it} N_{it} + (1-\tau_f) \Pi_{it}, \end{aligned} \tag{2}$$

where:

A_{it}, A_{it}^*	bunch of net domestic and net foreign assets held by a household,
S_t	nominal exchange rate,
P_t	consumer price index,
C_{it}	individual real consumption of household,
W_{it}	nominal wage,
N_{it}	labour supply,
i_t, i_t^*	nominal rate of returns on domestic and foreign assets,
Π_{it}	aggregated profit of firms,
ζ_t^*	risk premium,
τ_b	rate of benefits to households,
τ_w	personal income tax rate,
τ_f	corporate income tax rate,
τ_c	implicit tax rate on consumption (value added tax and excise tax).

Following Benigno (2001) or Schmitt-Grohe and Uribe (2003), we apply a hypothesis that a foreign interest rate realized on the international financial market differs with a borrower/lender position of a country. The interest rate is then a sum of “risk free” interest rate (i_t^*) and a “risk” premium (ζ_t^*). The later is a function of net foreign assets.[6] So, if the domestic economy is in the position of a net borrower then the domestic households are charged by the extra premium and therefore $\zeta_t^* > 0$, and vice versa.

Having specified the model, we can derive the optimal behaviour of households. An optimal decision constitutes from two parts: intertemporal and intratemporal optimization.

Intertemporal optimization: We solve an optimization problem where households seek an optimal allocation with respect to their limited resources. This relationship is not difficult to derive from the first order conditions (FOCs) of the maximization problem.

The Euler equation of consumption expresses the determinants of relative consumption depending on the risk aversion factor ψ_c and the habit formation parameter γ

$$1 = \beta E_t \left[\left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \left(\frac{C_{it+1} - H_{t+1}^c}{C_{it} - H_t^c} \right)^{-\psi_c} \right]. \tag{3}$$

In the case of labour supply, households offer labour with respect to the real wage and a consumption decision

$$N_{it}^{\psi_n} = \left[\frac{(1 - \tau_w + \tau_b) W_t}{(1 + \tau_c) P_t} \right] (C_{it} - H_t^c)^{-\psi_c} . \tag{4}$$

We can also derive determinants of the exchange rate in the form of an uncovered interest rate parity[7]

$$\frac{E_t(S_{t+1})}{S_t} = \frac{1 + i_t}{1 + i_t^* + \zeta_t^*}. \tag{5}$$

Intratemporal optimization: Households do not optimize consumption from intertemporal perspective only but they also consider which goods and services to consume, whether those domestically produced or imported ones. They try to minimize consumption expenses, so the important determinants are relative prices of goods in the market. An optimization problem may be formalized as follows

$$\min_{C_t^d, C_t^m} P_t C_t = P_t^d C_t^d + P_t^m C_t^m, \tag{6}$$

[6] The risk premium is a function of NFA/GDP. From a balance of payments we know that $\Delta NFA_t \approx CA_t + FA_t$, where CA_t stands for the current account and FA_t the financial account. Since many transactions from the financial account are sometimes sterilized, we decided to ignore this component of net financial assets in the model. Although it is a rough approximation, it can be shown that the main relations are still preserved.

[7] The way how to derive the uncovered interest rate parity in DSGE models is from the optimization problem of households. This is also the reason why the risk premium occurs in households' budget constraint. Otherwise we would not be able to consistently derive exchange rate including risk premium.

subject to a consumption bundle

$$C_t = \left[(1-\mu_{cm})^{1-\theta_c}\left(C_t^d\right)^{\theta_c} + (\mu_{cm})^{1-\theta_c}\left(C_t^m\right)^{\theta_c}\right]^{\frac{1}{\theta_c}}, \tag{7}$$

where:

C_t	aggregate consumption,
C_t^d	consumption of domestically produced goods,
C_t^m	consumption of imported goods,
P_t	consumption price index,
P_t^d	price of domestically produced goods for consumption,
P_t^m	price of imported goods for consumption,
θ_c	parameter of substitution between domestic and imported goods,
μ_{cm}	weight of the imported consumption in the bundle.

Both consumption bundles are assume to be Dixit-Stiglitz consumption aggregates of individual consumption goods

$$C_t^d = \left[\int_0^1 \left(C_{jt}^d\right)^{\theta_d} \mathrm{d}j\right]^{\frac{1}{\theta_d}}, \qquad C_t^m = \left[\int_0^1 \left(C_{jt}^m\right)^{\theta_m} \mathrm{d}j\right]^{\frac{1}{\theta_m}}, \tag{8}$$

where θ_d and θ_m are parameters of substitution among goods in baskets of domestically produced and imported goods. Households decide for domestic and foreign goods respectively, in compliance with the weight of respective goods in their consumption and with their substitutability. A parameter of substitution θ_c is defined using the elasticity of substitution in consumption (σ_c) as $\theta_c = 1 - 1/\sigma_c$.[8] We may derive another key macroeconomic relationship, such as aggregate consumer price index P_t, which depends on prices of domestic and foreign goods weighted by their shares in consumption

$$P_t = \left[(1-\mu_{cm})\left(P_t^d\right)^{1-\sigma_c} + \mu_{cm}\left(P_t^m\right)^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}}, \tag{9}$$

and also demand for the two consumption categories

$$C_t^d = (1-\mu_{cm})C_t\left(\frac{P_t^d}{P_t}\right)^{-\sigma_c}, \qquad C_t^m = \mu_{cm}C_t\left(\frac{P_t^m}{P_t}\right)^{-\sigma_c}. \tag{10}$$

The latter is determined by the household's decision about the level of current consumption, weights of domestic and imported goods in consumption, their substitutability and relative prices.

2.2 Firms

There are assumed to be three types of firms, indexed by $j \in [0,1]$, operating in the domestic economy: intermediate-goods producers (hereafter only producers), final-goods producers (retailers), and import firms (importers). In order to improve the

[8] We derive parameters θ_d and θ_m analogically.

performance of a supply side of the model, it seems to be necessary to incorporate some form of nominal and real rigidities into the model.[9] We use a simple Cobb-Douglas production function, extended for adjustment costs for input factors. Only for the sake of simplicity, we assume quadratic (symmetric) adjustment costs.[10] An example of adjustment cost function is as follows

$$\Upsilon_t^x = \frac{\upsilon}{2}\left(\frac{\Delta X_t}{X_{t-1}} - \dot{x}\right)^2, \tag{11}$$

where $X_t \in \{L_t, M_t\}$ denotes employment and imports, and $\dot{x} \in \{\dot{l}, \dot{m}\}$ means their steady state growth rates, respectively, υ is a constant term. An interested reader is referred to Hamermesh and Pfann (1996), for a formal explanation and excellent discussion about adjustment cost functions.

Retailers

Retail firms are assumed to behave in a perfect competitive environment.[11] They buy intermediate goods from producers, aggregate and sell them to households or government in the domestic or foreign economy. The competitive environment causes that prices of goods are set by producers and retailers can optimize only according to quantities Q_{jt}. Maximization a profit function is thereby equivalent to minimization a cost function due to a zero profit margin condition. Each retail firm tries to minimize the following cost function

$$\min_{Q_{jt}} \quad P_t^d Q_t = \int_0^1 P_{jt}^d Q_{jt} \mathrm{d}j, \tag{12}$$

where:

P_{jt}^d	individual price of a good j,
P_t^d	aggregate price index,
Q_{jt}	individual intermediate good j,
Q_t	aggregated output.

subject to a bundle generated by the Dixit-Stiglitz aggregate

$$Q_t = \left[\int_0^1 (Q_{jt})^{\theta_q} \mathrm{d}j\right]^{\frac{1}{\theta_q}}, \tag{13}$$

[9]Another advantage of using this approach is in an explanation of some issues known from the RBC literature. A textbook example is rigidity of employment during the economic recession despite rigidity in nominal wages. An explanation can be found in real rigidity: especially in lower efficiency of new (additional) unit of workers (input factor) which can be captured, at least to some extent, by above mentioned costs of adjustment functions.

[10]In the next version of HUBERT we would like to respecify our adjustment costs to be simple nonlinear/asymmetric functions derived from micro-surveys provided they are available.

[11]A perfect competition assumption is used for the convenience since it simplifies the derivation of an aggregate price index, see below.

where θ_q is a parameter of substitution among goods in the whole basket. From the FOC follows a demand function for an individual good Q_{jt} given by

$$Q_{jt} = Q_t \left(\frac{P^d_{jt}}{P^d_t} \right)^{-\sigma_q}, \tag{14}$$

and the aggregate price index

$$P^d_t = \left[\int_0^1 \left(P^d_{jt} \right)^{1-\sigma_q} \mathrm{d}j \right]^{\frac{1}{1-\sigma_q}}, \tag{15}$$

where σ_q is a price elasticity.

Producers

We assume that there is a continuum of monopolistically competitive intermediate-goods firms producing a differentiated output Q_{jt} using a simple production function with costs of adjustment. Due to monopolistic competition, producers are given some power in price setting behaviour. Following Calvo (1983), we assume that firms are allowed to reset their output prices only after receiving a random signal. The probability of receiving such the signal at time t is $(1-\xi_p) \in [0,1]$. Moreover, in line with Gali and Gertler (1999) we assume that a fraction of firms μ_p, allowed to reoptimize their price at time t, uses some simple rule for price updating: $P^b_{jt} = (1+\pi_{t-1})\tilde{P}_{t-1}$, where $\tilde{P}_{t-1}$ is a lagged optimal price of their competitors. Loosely speaking, a part of firms that can reoptimize output prices is backward-looking and the rest of them is forward-looking. Those firms that cannot reoptimize are assumed to follow a simple price setting rule: $P_{jt} = P_{jt-1}$.

Each producer, indexed by $j \in [0,1]$, tries to maximize the following profit function

$$\max_{\{P^d_{jt}, L_{jt}, M_{jt}\}} \quad \Pi^d_{j0} = E_t \sum_{t=0}^{\infty} (\beta \xi_p)^t (P^d_{jt} - MC^d_t) Q_{jt}, \tag{16}$$

subject to a very simple production function in the form

$$Q_{jt} = Z_t L^{\alpha}_{jt} M^{1-\alpha}_{jt} - \Upsilon^l_t L_{jt} - \Upsilon^m_t M_{jt}, \tag{17}$$

where:

P^d_{jt}	individual price of a good j,
MC^d_t	domestic marginal costs,
Q_{jt}	individual output of a good j,
L_{jt}	employment,
M_{jt}	imported goods for consumption,
Z_t	technological progress,
α	production function parameter,
Υ^l_t, Υ^m_t	adjustment cost functions.

From the standard FOCs result the optimal demand for labour and imports in the form as follows

$$\frac{W_t}{P_{jt}^d} = \frac{\alpha Q_{jt}}{L_{jt}} - \Upsilon_t^l - \upsilon\left(\frac{\Delta L_{jt}}{L_{jt-1}} - \dot{l}\right)\frac{L_{jt}}{L_{jt-1}} + \upsilon\beta\xi_p E_t\left[(1+\pi_{t+1})\left(\frac{\Delta L_{jt+1}}{L_{jt}} - \dot{l}\right)\left(\frac{L_{jt+1}}{L_{jt}}\right)^2\right], \tag{18}$$

$$\frac{S_t P_t^*}{P_{jt}^d} = \frac{(1-\alpha)Q_{jt}}{M_{jt}} - \Upsilon_t^m - \upsilon\left(\frac{\Delta M_{jt}}{M_{jt-1}} - \dot{m}\right)\frac{M_{jt}}{M_{jt-1}} + \\ + \upsilon\beta\xi_p E_t\left[(1+\pi_{t+1})\left(\frac{\Delta M_{jt+1}}{M_{jt}} - \dot{m}\right)\left(\frac{M_{jt+1}}{M_{jt}}\right)^2\right], \tag{19}$$

where P_t^* represents the foreign consumer price index and $S_t P_t^*$ represents marginal costs of imported intermediate goods.[12]. It is worth nothing that a demand for our exports can be derived just by reverting the above derived import FOC in (19).

The FOC for producers allowed to reoptimize their prices is as follows

$$P_{jt}^f = \left(\frac{\sigma_q}{\sigma_q - 1}\right)\left[\frac{E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i MC_{t+i}^d Q_{jt+i}}{E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i Q_{jt+i}}\right], \tag{20}$$

where P_{jt}^f is the optimal price of a forward-looking producer, θ_q is the parameter of substitution among goods in a basket, MC_t^d is the marginal costs specification, and Q_{jt} is the output quantity. The aggregate price index is a function of newly set prices and updated prices from the previous period

$$P_t^d = \left[\int_0^1 \left(P_{jt}^d\right)^{1-\sigma_q} \mathrm{d}j\right]^{\frac{1}{1-\sigma_q}} = \left[(1-\xi_p)(\tilde{P}_t^d)^{1-\sigma_q} + \xi_p (P_{t-1}^d)^{1-\sigma_q}\right]^{\frac{1}{1-\sigma_q}}, \tag{21}$$

where σ_q is the elasticity of substitution among goods, and $\tilde{P}_t^d$ is a function of forward-looking price setters using P_{jt}^f as the optimal price and backward-looking price setters using an optimal price P_{jt}^b.

Importers

The Czech economy, as a small open economy, is in the position of a price taker in foreign trade markets. Importers purchase foreign goods at given foreign prices (marginal costs)

[12]Only for simplicity we assume that firms have a direct link to foreign producers.

and turn it into differentiated import goods used for consumption. We assume that each importing firm, indexed by $j \in [0,1]$, tries to maximize a profit function given by

$$\max_{\{P^m_{jt}\}} \quad \Pi^m_{j0} = E_t \sum_{t=0}^{\infty} (\beta\xi_p)^i (P^m_{jt} - MC^m_t) C^m_{jt}, \tag{22}$$

subject to a very simple demand function in the form

$$C^m_{jt} = C^m_t \left(\frac{P^m_{jt}}{P^m_t} \right)^{-\sigma_m}, \tag{23}$$

where:

P^m_{jt}	individual price of imported good j,
P^m_t	aggregate price of imported goods,
P^m_t	price of imports,
MC^m_t	marginal costs of imported goods,
C^m_{jt}	individual imported good j,
C^m_t	imported goods,
σ_m	elasticity of substitution among imported goods.

The FOC for importers, which is an analogue to the FOC of domestic producers, results the following aggregate price index of imported goods

$$P^m_t = \left[\int_0^1 \left(P^m_{jt}\right)^{1-\sigma_m} \mathrm{d}j \right]^{\frac{1}{1-\sigma_m}} = \left[(1-\xi_p)(\tilde{P}^m_t)^{1-\sigma_m} + \xi_p (P^m_{t-1})^{1-\sigma_m} \right]^{\frac{1}{1-\sigma_m}}, \tag{24}$$

where the notation is identical to the previous section.

2.3 Labour market

Each household, indexed by $i \in [0,1]$, is assumed to supply a differentiated type of labour to intermediate producers. Imperfect substitution of labour provides some monopoly power to households in wage negotiation. Following Erceg, Henderson and Levin (1999), we assume that households can negotiate wage only after receiving some random signal. The probability of receiving such a signal at time t is $(1-\xi_w) \in [0,1]$. Since wages are set in the form of staggered contracts, each households reoptimize its wage rate by maximizing utility function in equation (1) with respect to W_{it} and subject to a standard labour demand function. The resultant FOC gives

$$\tilde{W}_{it} = \left(\frac{-\sigma_l}{1+\tau_c} \right) \left[\frac{E_t \sum_{j=0}^{\infty} (\beta\xi_w)^j U_{nt+j} N_{it+j}}{E_t \sum_{j=0}^{\infty} (\beta\xi_w)^j (1-\tau_w+\tau_b) U_{ct+j} N_{it+j}/P_{t+j}} \right], \tag{25}$$

where:

$\tilde{W}_{it}$	newly negotiated wage,
U_{nt}	marginal disutility of labour,
U_{ct}	marginal utility of consumption,
θ_l	parameter of substitution.

Those households who cannot re-set their wages follow a simple wage rule: $W_{it} = W_{it-1}$, it means there is no indexation of wages. The aggregate wage is a function of newly negotiated wage and updated wage from the previous period

$$W_t = \left[\int_0^1 (W_{it})^{1-\sigma_l}\,\mathrm{d}i\right]^{\frac{1}{1-\sigma_l}} = \left[(1-\xi_w)(\tilde{W}_t)^{1-\sigma_l} + \xi_w (W_{t-1})^{1-\sigma_l}\right]^{\frac{1}{1-\sigma_l}}, \quad (26)$$

where σ_l is the elasticity of labour substitution.

2.4 Monetary policy

Conducting of monetary policy has undergone through two different strategies recently in the Czech Republic. The first one, adopted and carried out during a transformation period, was based on targeting specific monetary aggregates. However, this strategy was not very successful especially due to a fixed exchange rate attracting foreign speculative capital. After exchange regime fluctuations in 1997, the Czech National Bank (CNB) adopted a new strategy for monetary policy based on inflation targeting. Roughly speaking, the main point of this approach is to set the main instrument of the central bank according to key macroeconomic variables such as inflation, output gap and possibly other relevant variables, in order to achieve and maintain a price stability.

We approximate bank's behavior by the extended Taylor rule, developed by Taylor (1993) and discussed by Svensson (1998).[13] The rule takes the following form

$$i_t = (1-\phi_i)[\bar{i} + \lambda_\pi \hat{\pi}_t + \lambda_y \hat{y}_t] + \phi_i i_{t-1} \quad (27)$$

where

i_t	is short-term nominal interest rate,
$\bar{i}$	steady state value of short-term interest rate,
$\hat{\pi}_t$	deviation of inflation rate from its steady state (target) value,
$\hat{y}_t$	output gap,
λ_y	output gap weight,
λ_p	inflation weight,
ϕ_i	interest rate smoothing parameter.

According to Srour (2001), there are many reasons for interest rate smoothing. First, the behaving of the central bank is important for investors and smoothing of interest rates can reduce volatility of a term premium and therefore volatility of long-term

[13] We do not consider an inflation forecast-based rules since we have witnessed rather accommodative approach of monetary policy recently.

interest rates and other financial market instruments. Second, the central bank has usually limited information about the shocks hitting the economy. Third, many shocks are serially correlated.

According to Levin et al. (1998), simple monetary policy rules with a high degree of interest rate smoothing ($\phi_i \to 1$) are also surprisingly robust against model uncertainty and misspecification. Unfortunately, this is probably a characteristic feature for large closed economies only. Coté et al. (2002) find that the most robust rule is the original Taylor rule ($\phi_i \to 0$) for small open economy. Much worse, Natvik (2006) showed that extending a DSGE model for a fiscal block can lead to a serious determinacy problem. From this point of view, a cautionary note should be made for straightforward application of Taylor rules.

2.5 Fiscal policy

Although we can find a large body of the literature analyzing different issues of monetary policy using DSGE models, fiscal policy applications are rather rare and leading often to controversial results. For instance, one of the most daunting issue of fiscal policy is related to contradictionary effects of government expenditures on key macroeconomic variables such as employment and/or output coming from the empirical (VAR) and structural (DSGE) models. VAR models usually predict a positive effect of government expenditures on both output and employment which is in sharp contrast to the main findings from DSGE models, see Fatás and Mihov (2001), Gali et al. (2007), or Coenen and Straub (2005) for details. However, we strongly believe that fiscal policy can be a very efficient and powerful tool for economic policy especially due to many different instruments that can be used. So, the main purpose of this section is to present a simple fiscal policy rule closing the model.[14] Although this task may seem quite simple, it is by no means easy to introduce even apparently unsophisticated rule into the model. An ad-hoc fiscal and monetary rules may lead to unintended implications, see Ascari and Rankin (2007) for details.

2.5.1 Government Budget

Revenues of the government budget GR_t consist of the following three categories

$$GR_t = PIT_t + CIT_t + VAT_t + EXCISE_t, \tag{28}$$

[14]Ongoing research at the Ministry of Finance focuses on the above mentioned issue of government expenditures. Some improvements of the fiscal block will be implemented into the next version of the model.

where:

GR_t	total government revenues,
PIT_t	personal income tax revenues,
CIT_t	corporate income tax revenues,
VAT_t	value added tax revenues.
$EXCISE_t$	excise revenues.

A significant part of taxes are those from income. Revenues from a personal income tax revenues PIT_t depend on implicit personal income tax rate τ_w and on a wage bill W_tL_t.[15] The corporate income tax revenues CIT_t are determined by the implicit rate τ_f and the profit Π_t. Important part of revenues is also value added tax revenues VAT_t and excise tax revenues $EXCISE_t$ that are imposed on (nominal) consumption P_tC_t.[16]

Government expenditures GE_t consist of two important groups of expenditures: government consumption and different kinds of social benefits. Government consumption G_t includes purchases of individual and collective goods. Social security expenditures G_t^s consist of retirement expenditures, sickness expenditures, unemployment benefits and other social security expenditures. So, the government expenditures take the following form

$$GE_t = G_t + G_t^s, \tag{29}$$

GE_t	total government expenditures,
G_t	government expenditures,
G_t^s	social benefit.

Having derived the main equations for revenues (28) and expenditures (29) it is easy to get identity for deficit

$$D_t = GE_t - GR_t, \tag{30}$$

and also government debt

$$B_t = D_t + (1 + i_{t-1})B_{t-1}, \tag{31}$$

where:

GR_t	total revenues,
GE_t	government expenditures,
B_t	total government debt,
D_t	government deficit,
i_t	nominal interest rate.

2.5.2 Fiscal policy rule

In the previous section we defined main budgetary items that are of the most importance when analyzing public finances. Our main goal is to derive fiscal rule to close the model properly. Two important questions must be answered when doing so. First, what

[15] All social security payments are included in PIT_t.

[16] We do not distinguish explicitly VAT and EXCISE tax rates, only for simplicity.

will be a reference variable activating the fiscal policy rule - debt, deficit, or both. Second issue concerns an instrument that should be adjusted by the rule. Unfortunately, there is no clear evidence in the economic literature which instrument should play the main role. In general, most analysis rely on tax rules where fiscal policy rectifies the debt dynamics by changes in tax rates. Unluckily various difficulties are related with introducing tax rules into the model (an optimal taxation problem, omitted interactions with monetary policy and internal consistency of the model).

In our view, the best way how to deal with these difficulties is to introduce an expenditure fiscal rule. First, the expenditure based rule is much easier. We do not need to arbitrarily decide which tax rate should be adjusted. Second, changes in tax rates require a change in legislation which can be very inflexible. Contrary, government expenditures may be adjusted quite promptly. Third, changes in taxation has an impact on relative prices. Our fiscal policy rule is based on an assumption of a balanced primary government budget (zero primary deficit) in equilibrium.[17] Formally, the rule is given by the following equation

$$G_t = (1-\phi_g)\bar{G}_{t-1} + \phi_g G_{t-1}, \tag{32}$$

where:

G_t	government expenditures,
$\bar{G}_t$	equilibrium expenditures.

Equilibrium expenditures $\bar{G}_t$ are derived from equation (30) under the following condition: $D_t = GE_t - GR_t = 0$. One period lag of equilibrium government expenditures $\bar{G}_{t-1}$ in the fiscal rule reflects the information delay. The parameter ϕ_g reflects a speed of convergence of public finances.

2.6 World

The Czech economy can be characterized as a small open and raw materials dependent economy. For this reason a transmission of world economy shocks (oil price, etc.) into the Czech key macroeconomic variables is of the crucial importance for economic policy. Therefore, we decided to incorporate a small model of the EU economy into our model.

There are two alternatives how to deal with this issue. The first solution is to build up a small structural model of the EU economy. The second one is to follow results form simple VAR models, see Lindé et al. (2004) for an example. VAR models are easy to estimate and operate with but they can give us confusing results without any sign/parameter restrictions. On the other hand, a small calibrated structural model can have a problematic fit with actual data and it is by no means easy to calibrate. All in all, we decided to rely on a small structural model due to Cho and Moreno (2003),

[17] We use primary deficit in order to avoid the coincident restriction of public finances implied by the monetary restriction through interest rate payments, and vice versa.

which ia a simplified version of Smets and Wouters (2002) model. The world model captures the behaviour of three European agents: households, firms, and government: households are assumed to maximize a simple utility subject to budget constraint (analogically to equations (1) and (2)); monopolistically competitive producers maximize a profit and set output prices following the Calvo price setting mechanism (analogically to equations (20) and (21)); the government is assumed to determine a short-term interest rate following the Taylor rule (analogically to equation (27)).

Due to the importance of the link between the Czech and European economy, a detailed comparison of the structural and empirical modelling of the European economy is the subject of a special working paper. Only for interest, we present a preliminary comparison of impulse-response analysis of three basic shocks from a small structural model and VAR(1) counterpart in Section 4. From our findings result the cautionary use of unrestricted VAR model, see Figure 5 in Section 4.3.

3 Solution of the model

3.1 Log-linearization

The derived FOCs are usually quite complicated nonlinear functions in both parameters and variables. To deal with these non-linear equilibrium conditions can be very tedious (from numerical point of view) and the solution of the model can be sensitive to a very small perturbation of the variables. Therefore, the model equations are usually linearized using a linear or nonlinear approximation around steady states. We use a simple linear approximation. Let's assume that the equilibrium condition is given by the the following (nonlinear) function[18]

$$x_t = f(x_{t-1}, z_t), \tag{33}$$

where x denotes an endogenous variable and z an exogenous variable. Then, the function $f(x_{t-1}, z_t)$ can be approximated as follows

$$x_t \approx f(\bar{x}, \bar{z}) + \left(\frac{\partial f(\bar{x}, \bar{z})}{\partial x_{t-1}}\right)(x_{t-1} - \bar{x}) + \left(\frac{\partial f(\bar{x}, \bar{z})}{\partial z_t}\right)(z_t - \bar{z}), \tag{34}$$

where $\bar{x}$ and $\bar{z}$ denote steady states of variables. Provided that all variables are in logs ($x \equiv \log X$, $z \equiv \log Z$) then $\partial f(\cdot)/\partial x$ and $\partial f(\cdot)/\partial z$ denote elasticities. However, it is worth noting that when doing the approximation we have to define properly steady state values of the model which is usually a little bit tricky, see next section for some examples.

The linear approximation method is easy to calculate and many estimation methods are available. Moreover, unobserved model variables can easily be generated by a basic Kalman filter method. The nonlinear approximation methods can be much more complicated and time consuming to apply, especially in larger models. So, the question is whether nonlinear approximation methods lead to significant improvements in general. The answer is case dependent, unfortunately. For instance, DeJong and Dave (2007) show, using a standard RBC model, that the performance of linear and nonlinear approximation methods in matching empirical moments is pretty much the same. Fernández-Villaverde and Rubio-Ramirez (2005) focus on the estimation of parameters of a simple RBC model under linear and nonlinear approximation. They end up with two findings. First, point estimates of unknown model parameters are comparable regardless the approximation method. Second, the nonlinear approximation leads to more efficient estimates. Unfortunately, there is no single evidence, at least to our best knowledge, from medium/large models. Therefore, nonlinear approximation methods are usually used only for specific reasons such as deriving term premia, see Rudebush and Swanson (2007). From all the above mentioned reasons we use only a simple linear approximation of model equations.

[18] We also assume that the function $f(\cdot)$ has continuous derivatives.

3.2 Steady-states

To determine steady state values of variables means to determine long-run dynamic properties of the core model. We suppose that all variable on the equilibrium path are either constant (stationary variable) or exponentially growing with the constant growth rate (nonstationary variables).

Determination of steady state values can be illustrated using the following examples. The steady state inflation rate, denoted as $\bar{\pi}$, is not derived explicitly from the static version of the NK Phillips curve but from the inflation target of the Czech National Bank. According to CNB's strategy, the inflation rate target is set to be 3 % from January 2006 (and 2 % from January 2010). However, this approach can simply lead to inconsistency of the model provided that a given inflation target is incorrectly set.

The steady state growth rate of domestic output, denoted as $\dot{q}$, is derived explicitly from a given production function in Section 2.2. Clearly, from the log-linear production function follows that $\dot{q} = \dot{z}/\alpha + \dot{n}$, where $\dot{z}$ is the steady state growth rate of (exogenous) technology progress, and $\dot{n}$ denotes the steady state growth rate of population. Both quantities are calibrated according to Czech business cycle facts.

3.3 Model

Using the log-linearization we are able to rewrite the model into following system of equations that can be solved easily. We sort the equation to show how the main macroeconomic variables are determined. The equations are simplified using reduced-form parameters denoted by ω_{ij}, where the index i denotes a particular equation and j a given variable. We also use a simplified notation for shocks.

Output

Following equations illustrate how the model derives the determinants of the output – consumption, government expenditures and net exports. We assume in this version investments to be a part of the consumption expenditures. Consumption itself stems from domestic producers or from imports, while factors of their mutual share are their relative prices. Firms output ($\hat{q}_t$) is produced using the technology, labour and imported sources.

$$\hat{y}_t = \omega_{yc}\hat{c}_t^d + (1-\omega_{yc})\hat{g}_t + \omega_{yx}(\hat{e}_t - \hat{m}_t), \tag{35a}$$

$$\hat{c}_t = \omega_{cc}\hat{c}_{t-1} + (1-\omega_{cc})E_t(\hat{c}_{t+1}) - \omega_{ci}E_t(\hat{i}_t - \hat{\pi}_{t+1}) + u_t^c, \tag{35b}$$

$$\hat{g}_t = \omega_{gd}(\hat{d}_{t-1} - \hat{p}_t) + \omega_{gg}\hat{g}_{t-1} + u_t^g, \tag{35c}$$

$$\hat{e}_t = (1-2\omega_{ee})[\hat{q}_t^* - (\hat{p}_t^e - \hat{p}_t^* - \hat{s}_t)] + \omega_{ee}\hat{e}_{t-1} + \omega_{ee}E_t(\hat{e}_{t+1}), \tag{35d}$$

$$\hat{m}_t = (1-2\omega_{mm})[\hat{q}_t - (\hat{p}_t^* + \hat{s}_t - \hat{p}_t^d)] + \omega_{mm}\hat{m}_{t-1} + \omega_{mm}E_t(\hat{m}_{t+1}). \tag{35e}$$

$$\hat{c}_t^d = \hat{c}_t - \sigma_c(\hat{p}_t^d - \hat{p}_t), \tag{36a}$$
$$\hat{c}_t^m = \hat{c}_t - \sigma_c(\hat{p}_t^m - \hat{p}_t). \tag{36b}$$

$$\hat{q}_t = \hat{z}_t + \alpha\hat{l}_t + (1-\alpha)\hat{m}_t, \tag{37a}$$
$$\hat{z}_t = \omega_{zz}\hat{z}_{t-1} + v_t^z. \tag{37b}$$

Prices

The model works with different price indexes defined with following relations. Weighted prices of domestic and imported production are source of consumer price index ($\hat{\pi}_t$). These components – price indexes of domestic, imported and exported production – having both forward and backward looking component are derived using a concept of marginal costs.

$$\hat{\pi}_t = (1-\omega_{cm})\hat{\pi}_t^d + \omega_{cm}\hat{\pi}_t^m, \tag{38a}$$
$$\hat{\pi}_t^d = \omega_{pc}\hat{mc}_t^d + \omega_{pp}\hat{\pi}_{t-1}^d + (1-\omega_{pp})E_t(\hat{\pi}_{t+1}^d), \tag{38b}$$
$$\hat{mc}_t^d = \alpha(\hat{w}_t - \hat{p}_t) + (1-\alpha)(\hat{mc}_t^m - \hat{p}_t) - \hat{z}_t + u_t^p, \tag{38c}$$
$$\hat{\pi}_t^m = \omega_{pc}\hat{mc}_t^m + \omega_{pp}\hat{\pi}_{t-1}^m + (1-\omega_{pp})E_t(\hat{\pi}_{t+1}^m), \tag{38d}$$
$$\hat{mc}_t^m = \hat{p}_t^* + \hat{s}_t, \tag{38e}$$
$$\hat{\pi}_t^e = \omega_{pc}\hat{mc}_t^e + \omega_{pp}\hat{\pi}_{t-1}^e + (1-\omega_{pp})E_t(\hat{\pi}_{t+1}^e), \tag{38f}$$
$$\hat{mc}_t^e = \hat{mc}_t^d + \hat{s}_t. \tag{38g}$$

Labour market

The extent of a labour supply ($\hat{n}_t$) depends on households' decision while labour demand is ruled by firms. Wage on the labour market is a result of bargaining between employers and employees when only a segment of wage contracts is reset in each period of time.

$$\hat{w}_t = (1-2\omega_{ww})(\hat{y}_t - \hat{l}_t + \hat{p}_t) + \omega_{ww}\hat{w}_{t-1} + \omega_{ww}E_t(\hat{w}_{t+1}), \tag{39a}$$
$$\hat{n}_t = \omega_{nw}(\hat{w}_t - \hat{p}_t^c) - \omega_{nc}(\hat{c}_t - \gamma\hat{c}_{t-1}), \tag{39b}$$
$$\hat{l}_t = (1-2\omega_{ll})[\hat{q}_t - (\hat{w}_t - \hat{p}_t)] + \omega_{ll}\hat{l}_{t-1} + \omega_{ll}E_t(\hat{l}_{t+1}). \tag{39c}$$

Central bank and financial market

Currently the most important financial market variable is short term interest rate set by the central bank using an extended Taylor rule. Modified uncovered interest rate parity determines an exchange rate.

$$\hat{i}_t = \omega_{i\pi}\hat{\pi}_t + \omega_{iy}\hat{y}_t + \omega_{ii}\hat{i}_{t-1} + u_t^i, \tag{40a}$$
$$\hat{s}_t = \omega_{ss}[E_t(\hat{s}_{t+1}) + \hat{s}_{t-1}] - \omega_s\hat{ca}_t - \omega_{ss}(\Delta\hat{i}_t - \Delta\hat{i}_t^*) + u_t^s. \tag{40b}$$

Government

Besides very simple structure of government revenues and expenditures, important variables for fiscal policy are deficits and debts.

$$\hat{gr}_t = \omega_{gw}(\hat{w}_t + \hat{l}_t) + \omega_{gf}(\hat{p}_t + \hat{q}_t) + \omega_{gc}(\hat{c}_t + \hat{p}_t), \tag{41a}$$
$$\hat{ge}_t = \omega_{gc}(\hat{g}_t + \hat{p}_t) + \omega_{gs}(\hat{w}_t + \hat{n}_t), \tag{41b}$$
$$\hat{d}_t = \omega_{dd}(\hat{ge}_t - \hat{gr}_t), \tag{41c}$$
$$\hat{b}_t = \omega_{bd}\hat{d}_t + \omega_{bb}\hat{b}_{t-1}. \tag{41d}$$

Trade

Current account balance is used to describe an external balance. It is also an important factor of exchange rate development.

$$\hat{ca}_t = \omega_{yx}[\hat{e}_t - \omega_{mq}\hat{m}_t - (1-\omega_{mq})\hat{c}_t^m] + \omega_{yx}[\hat{p}_t^e - \hat{p}_t^m]. \tag{42}$$

World

The development of other economies, approximated by the EU in the model, has not negligible impact on domestic agents. Thus we derived a small model with results of an output, price level and interest rates.

$$\hat{y}_t^* = \omega_{yy}^*\hat{y}_{t-1}^* + (1-\omega_{yy}^*)E_t(\hat{y}_{t+1}^*) - \omega_{yi}^*E_t(\hat{i}_t^* - \hat{\pi}_{t+1}^*) + u_t^{y*}, \tag{43a}$$
$$\hat{\pi}_t^* = \omega_{pp}^*\hat{\pi}_{t-1}^* + (1-\omega_{pp}^*)E_t(\hat{\pi}_{t+1}^*) + \omega_{pc}^*\hat{y}_t^* + u_t^{p*}, \tag{43b}$$
$$\hat{i}_t^* = \omega_{i\pi}^*\hat{\pi}_t^* + \omega_{iy}^*\hat{y}_t^* + \omega_{ii}^*\hat{i}_{t-1}^* + u_t^{i*}. \tag{43c}$$

3.4 Estimation and calibration

A question about an estimation of the model is very closely related to a method of approximation. Provided we rely on just the linear approximation method, the whole set of estimation procedures is available. Probably the most popular estimation method currently used in the DSGE literature is the Bayesian ML estimation, see Schorfheide (2000), DeJong et al. (2000), Fernández-Villaverde and Rubio-Ramirez (2004) or Schorfheide et al. (2009). Despite many advances of this method, Bayesian ML procedures heavily depend on the specification of priors (beliefs): diffuse priors lead to ML results whereas very specific priors serve as high penalization of the likelihood function and thus differ from ML results. Moreover, there are other issues such as stochastic singularity or joint normality of errors that can complicate the use of Bayesian ML method, see Ireland (2004) for a discussion.

Although there is a large body of the literature about DSGE models, just a few papers focuses on comparison of different estimation procedure currently used. An exemption is Ruge-Murcia (2007) who focuses on the estimation of a simple RBC model using four different methods: ML, GMM, SMM, and II. From the results conclude that estimates obtained by SMM are better than those from MLE (without priors), but just a little bit worse than those from Bayesian MLE (with really good priors). Since we do not have very good beliefs about priors of model deep parameters, we decide to focus rather on the SMM based estimation.[19]

[19] A complete estimation will be available in the updated version of HUBERT.

3.5 Parameters

Parameters in our model are rather calibrated using other DSGE studies (especially those mentioned in Introduction) and also using a limited information estimation method.[20] Following table summarizes deep parameters of the model with values currently used in the model.

Table 1: Deep parameters of the model

parameter	**description**	**block**	**value**	**method**
	General parameters			
β	discount factor	H	0.99	calibrated
ψ_c	consumption utility (risk aversion)	H	2.00	calibrated
ψ_n	labour utility (risk aversion)	H	5.00	calibrated
γ	consumption habit	H	0.80	calibrated
α	production function parameter	F	0.50	calibrated
	Policy parameters			
λ_π	sensitivity on inflation	F	1.50	calibrated
λ_y	sensitivity on output gap	F	0.25	calibrated
	Weights			
μ_p	weight of backward-looking firms	F	0.50	calibrated
μ_{yc}	weight of consumption on GDP	F	0.80	calibrated
μ_{yx}	weight of export on GDP	F	0.75	calibrated
μ_{gw}	weight of PIT on gov. revenues	G	0.60	calibrated
μ_{gb}	weight of SB on gov. exp.	G	0.40	calibrated
	Autoregressive parameters			
ϕ_g	autoregressive parameter	G	0.70	calibrated
ϕ_i	autoregressive parameter	M	0.70	calibrated
ϕ_l	autoregressive parameter	LM	0.45	calibrated
ϕ_m	autoregressive parameter	F	0.40	calibrated
ϕ_e	autoregressive parameter	F	0.40	calibrated
ρ_z	autoregressive parameter	F	0.90	calibrated
	Probabilities			
1-ξ_p	probability of price reopt.	F	0.50	calibrated
1-ξ_w	probability of wage reopt.	L	0.50	calibrated

3.6 Solution

It is important to note that the model consists of both lagged variables (e.g. $\hat{c}_{t-1}$) and expected future endogenous variables (e.g. $E_t\hat{c}_{t+1}$). That means, the model consists of

[20]We expect a further analysis on this topic in a separate research study.

backward and forward-looking variables and the unique solution to this type of models does not have to be easy to find. But in general, we can rewrite the model into the matrix form as follows

$$\mathbf{G}\hat{\mathbf{y}}_t = \mathbf{F}E_t(\hat{\mathbf{y}}_{t+1}) + \mathbf{H}\hat{\mathbf{y}}_{t-1} + \mathbf{L}\hat{\mathbf{z}}_t, \tag{44a}$$

$$\hat{\mathbf{z}}_t = \mathbf{N}\hat{\mathbf{z}}_{t-1} + \mathbf{u}_t, \tag{44b}$$

where $\hat{\mathbf{y}}_t$ is a vector of endogenous variables, $\hat{\mathbf{z}}_t$ is a vector of exogenous variables or shocks, $\mathbf{G}, \mathbf{F}, \mathbf{H}, \mathbf{L}, \mathbf{N}$ are matrices of structural (unknown) parameters which are functions of deep parameters, and $\mathbf{u}_t$ is a vector of disturbances in the model. After having the model in the matrix form, we can focus on a closed-form solution given by

$$\hat{\mathbf{y}}_t = \mathbf{P}\hat{\mathbf{y}}_{t-1} + \mathbf{Q}\hat{\mathbf{z}}_t, \tag{45a}$$

$$\hat{\mathbf{z}}_t = \mathbf{N}\hat{\mathbf{z}}_{t-1} + \mathbf{u}_t, \tag{45b}$$

where $\mathbf{P}, \mathbf{Q}$ are reduced form parameters of the model but still nonlinear and complicated functions of deep parameters. Finally, we can rewrite the whole model in the compact form as follows

$$\hat{\mathbf{x}}_t = \mathbf{\Phi}\hat{\mathbf{x}}_{t-1} + \mathbf{\Theta}\boldsymbol{\varepsilon}_t, \tag{46}$$

which is in fact a VAR(1) model, where $\hat{\mathbf{x}}_t = (\hat{\mathbf{y}}'_t, \hat{\mathbf{z}}'_t)'$ and $\boldsymbol{\varepsilon}_t = (\mathbf{0}', \mathbf{u}'_t)'$ is a vector of errors. A macroeconomic model written in this form can be used simply both for predictions and policy analysis simulations. An interested reader is referred to Hamilton (1994), Juselius (2006), or Lutkepohl (2007) for more information about VAR models.

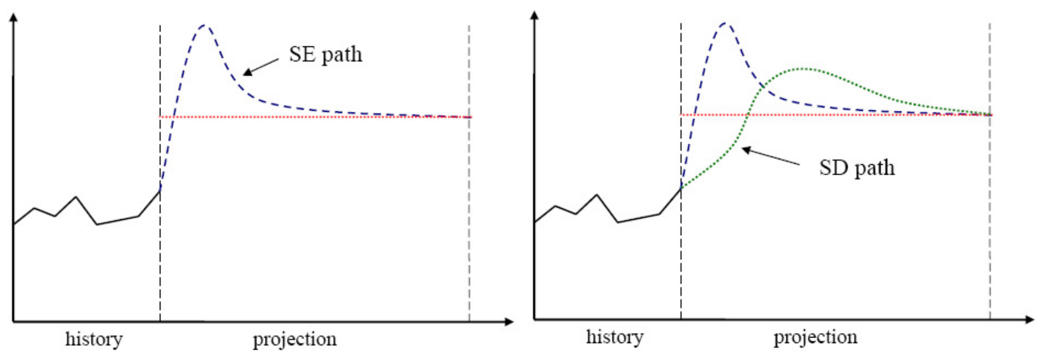
3.7 Forecasting

The equilibrium (core) part of the model is derived based on many simplifying assumptions such as constant model parameters, rational expectations, symmetric costs of adjustments, a simple log-linear approximation, etc. All these simplifications can significantly underestimate the short-run dynamics of endogenous variables. In addition, in many cases we are interested in forecasting variables that do not explicitly appear in the equilibrium (core) model. Harrison et al. (2005) solved this problem by using an error-correction approach which has many desirable properties. Recently, Schorfheide et al. (2009) propose a modification of the previous approach. We follow the original approach only for simplicity. So, the short-run dynamics of the model is given by

$$\Delta\mathbf{y}_t = \mathbf{\Gamma}\Delta\mathbf{y}_{t-1} + (\mathbf{I} - \mathbf{\Gamma})\Delta\mathbf{y}^*_t + \mathbf{\Psi}\Delta\mathbf{z}_t + \boldsymbol{\epsilon}_t, \tag{47}$$

where $\Delta\mathbf{y}_t$ denotes a vector of growth rates (or differences) of endogenous variables (e.g. inflation rate, etc.), $\Delta\mathbf{y}^*_t$ denote the short-run equilibrium path from the core model, $\Delta\mathbf{z}_t$ is a vector of exogenous variables (e.g. an oil price), or other variables useful for forecasting (e.g. one-off military expenditures), a higher degree of competition in the retail market, a change in the import intensity of a production, or long-term interest

Figure 2: Short-run equilibrium (SE) and short-run dynamic (SD) path



Source: Based on Harrison et al. (2005).

rates, etc.), and $\mathbf{\Gamma}$ and $\mathbf{\Psi}$ are matrices of (unknown) parameters.

There are at least two advantages of the above mentioned error-correction approach in forecasting. First, we can ensure the convergency of endogenous variables, so the stability of the model is well defined. Second, this specification allows us to capture specific dynamics in endogenous variables that is sometimes difficult to explain but that is important for predictions. A graphical explanation of the relationship between short-run equilibrium and short-run dynamics is depicted in Figure 2.

There are two nice textbook examples for which we can use the core/noncore system: a) modelling net inflation in the core model and consumer inflation outside the core system; b) modelling inventories that can be omitted from the core system but can be included into the noncore system used for forecasting.

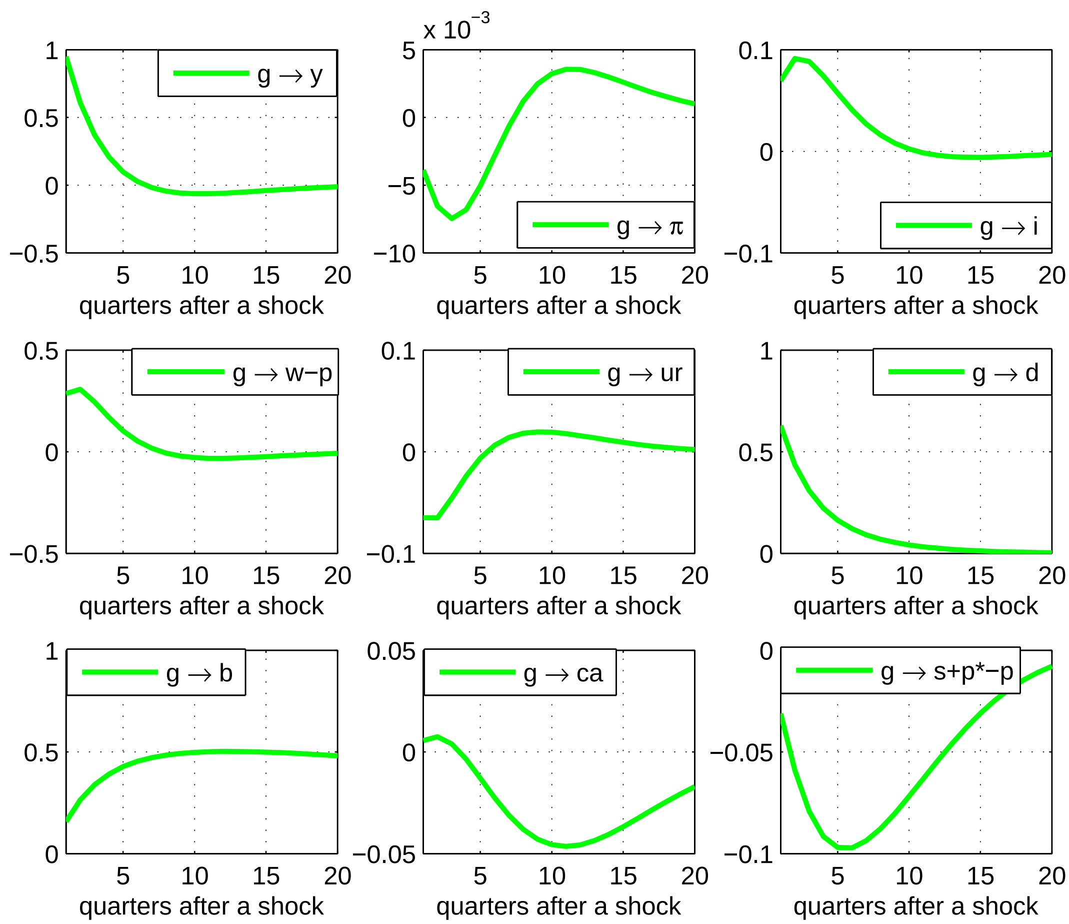
4 Simulations

We illustrate dynamic features of the model by impulse response functions of selected shocks. Only for simplicity, we assume two basic shocks: a monetary policy shock and a government expenditure shock. All shocks are expressed as percentage deviations from steady states of respective variables.

4.1 A fiscal policy shock in the Czech economy

A temporary increase in (real) government expenditures has an immediate and positive effect on real GDP. A higher demand is then translated in an additional demand for labour and therefore a temporary decrease in unemployment rate and increase in real wages.

Figure 3: IRFs of a temporary increase in government expenditures



In addition, higher demand altogether with exchange rate appreciation lead to worsening of a current account. On the other hand, due to a high degree of openness of the Czech economy, exchange rate appreciation reduces marginal costs and therefore inflation pressures of higher domestic demand.[21]

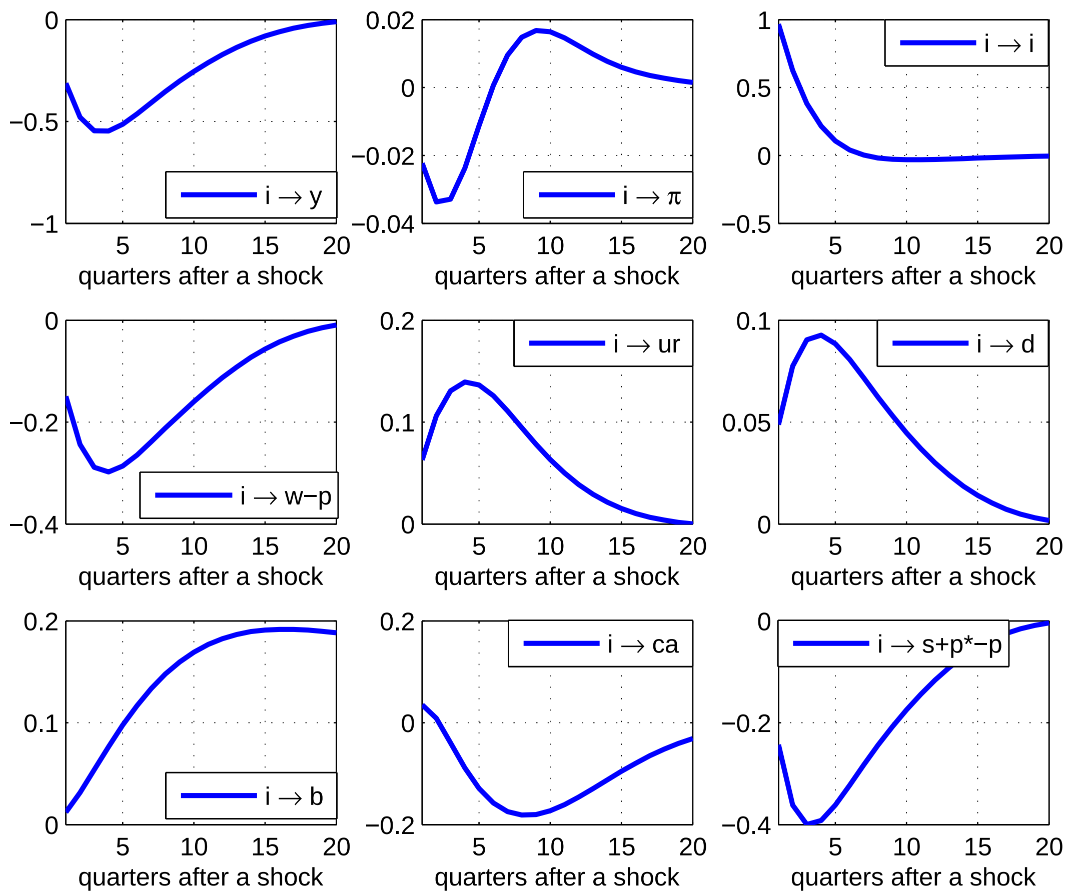
Higher government expenditures directly translate into the increasing deficit. Higher debt is a result of accumulated deficits altogether with higher nominal interest rates. It should be noted that an effect on debt can be very longlasting, depending especially on the specification of the fiscal policy rule in the model.

[21] However, it is worth noting that this effect is strongly parameter dependent. That means, the different specification of the exchange rate equation the different results for this simulation.

4.2 A monetary policy shock in the Czech economy

A temporary increase in the short-term nominal interest rate has a negative effect on real GDP through a consumption/investment channel, and subsequently leads to the higher unemployment rate. A positive interest rate differential between domestic and European rates leads to exchange rate appreciation which finally leads to worsening of a current account.

Figure 4: IRFs of a temporary increase in nominal interest rates



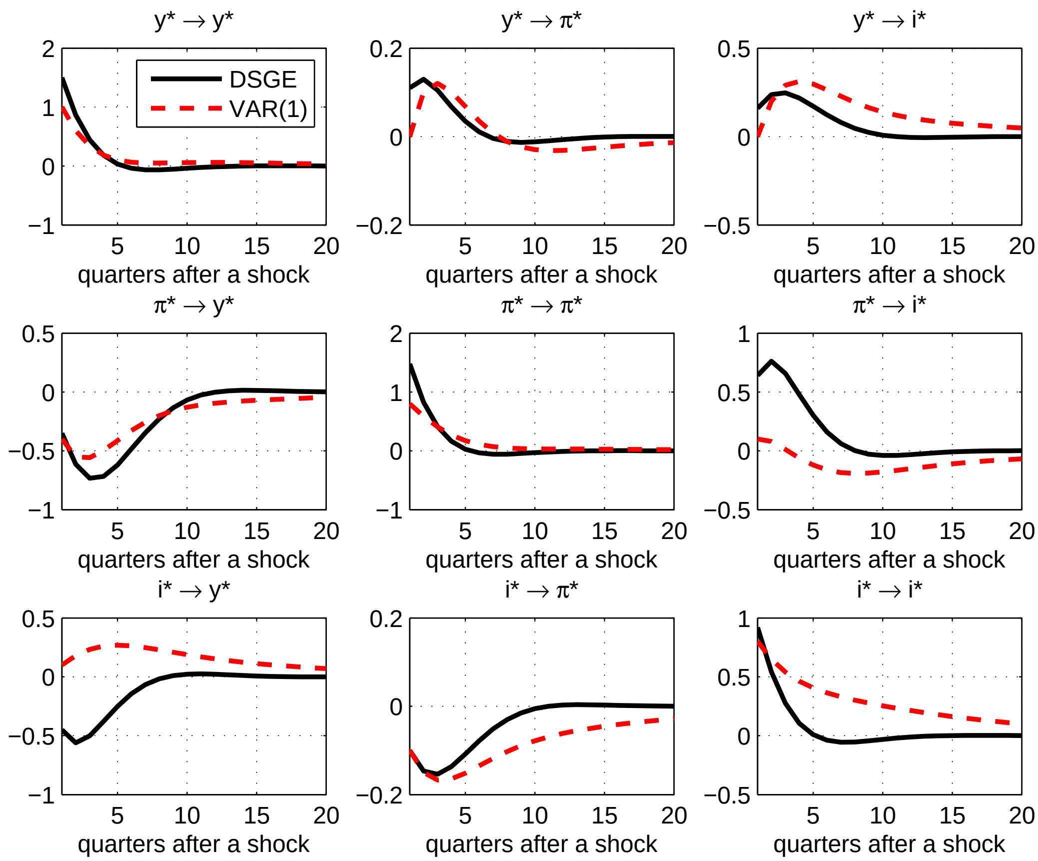
Higher nominal interest rates are transmitted into higher interest rate payments of government debt and altogether with higher primary deficit result in higher government debt.[22]

[22] Again, all the results are strongly affected by exchange rate specification.

4.3 Shocks in the EU economy

Since the Czech economy is a small open and commodity dependent economy, it is natural to focus on the question how the European shocks are transmitted in to the domestic economy. When doing so, we need some model of the European Economy. In the following figure we present basic results of an output shock, an inflation shock, and a monetary policy shock calculated from two different models: a simple (unrestricted) VAR model and a simple DSGE model discussed in Section 2.6. Although some impulse-response results seem to be encouraging, extreme caution should be exercised when applying simple VAR models.

Figure 5: IRFs of basic European shocks in two models



5 Conclusion and what next

From our preliminary experiments with HUBERT follow two important findings. First, although our model is rather a simplified version of a standard DSGE model and fully calibrated, it still provides a reasonably fit with data in more than 70 % of all variables. Second, from basic policy analysis simulations follow that the model could, after some modifications and extensions, provide an interesting policy analysis recommendations and answer many interesting questions.

In order to extend the scope of policy analysis simulations and/or to improve the performance of the model we propose the following modifications. First, we would like to introduce a liquidity constrained households that follow some "rule-of-thumb/hand-to-mouth" behaviour which seems to be very important for capturing effects of fiscal policy measures, see Furlaneto (2007) and Coenen and Straub (2005). Second, also firm specific capital and the price of capital definitely deserves more attention, see Woodford (2004), Woodford (2005) and Altig et al. (2005). Third, some additional labour market imperfections should be reflected by incorporating a matching function that can capture a limited matching between vacancies and unemployment in the economy, see Moyen and Sahuc (2004), Trigari (2004) and Stevens (2007). Forth, although the main intention is to concentrate on the fiscal policy, we would like to switch from external habit formation to internal habits which ensures us to incorporate long-term risk free interest rates into the model and no specific financial market/banking sector imperfections will be considered in the model. Fifth, we focus on implementation simple tax based fiscal policy rules into the model in order to provide standard policy analysis recommendations upon request. Finally, some important parameters that are rather difficult to calibrated will be estimated via to different estimation techniques: Bayesian MLE and SMM, see Ruge-Murcia (2007) for details.

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